

X-ray generation in Cosmic Sources

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Production of X-ray radiation

Requires highly energetic particles

In the cosmic setting, X-rays are produced by three main processes:

1. From very hot gas (Temperature > 1 million K), thermal bremsstrahlung emission, atomic transitions
2. From relativistic electrons streaming through magnetic fields, synchrotron emission
3. Compton Scattering of low-energy radiation field by energetic electrons

Classical radiation theory

Electromagnetic fields of moving charges

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \left[\frac{(\hat{n} - \vec{\beta})(1 - \beta^2)}{\kappa^3 R^2} \right] + \frac{q}{4\pi\epsilon_0 c} \left[\frac{\hat{n}}{\kappa^3 R} \times \left\{ (\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}} \right\} \right]$$

$$\vec{H}(\vec{r}, t) = \sqrt{\frac{\epsilon_0}{\mu_0}} \left[\hat{n} \times \vec{E}(\vec{r}, t) \right]$$

where

$$\kappa = 1 - \frac{1}{c} \hat{n}(t') \cdot \vec{u}(t')$$

$$\vec{R} = \vec{r} - \vec{r}_0(t')$$

$$\hat{n} = \vec{R}/R$$

$$c(t - t') = R(t')$$

An accelerated electric charge emits e.m. radiation

Uniformly moving charge:

Coulomb Field : $\sim 1/r^2$ from the *current* location of the charge

Accelerated charge:

J.J. Thomson

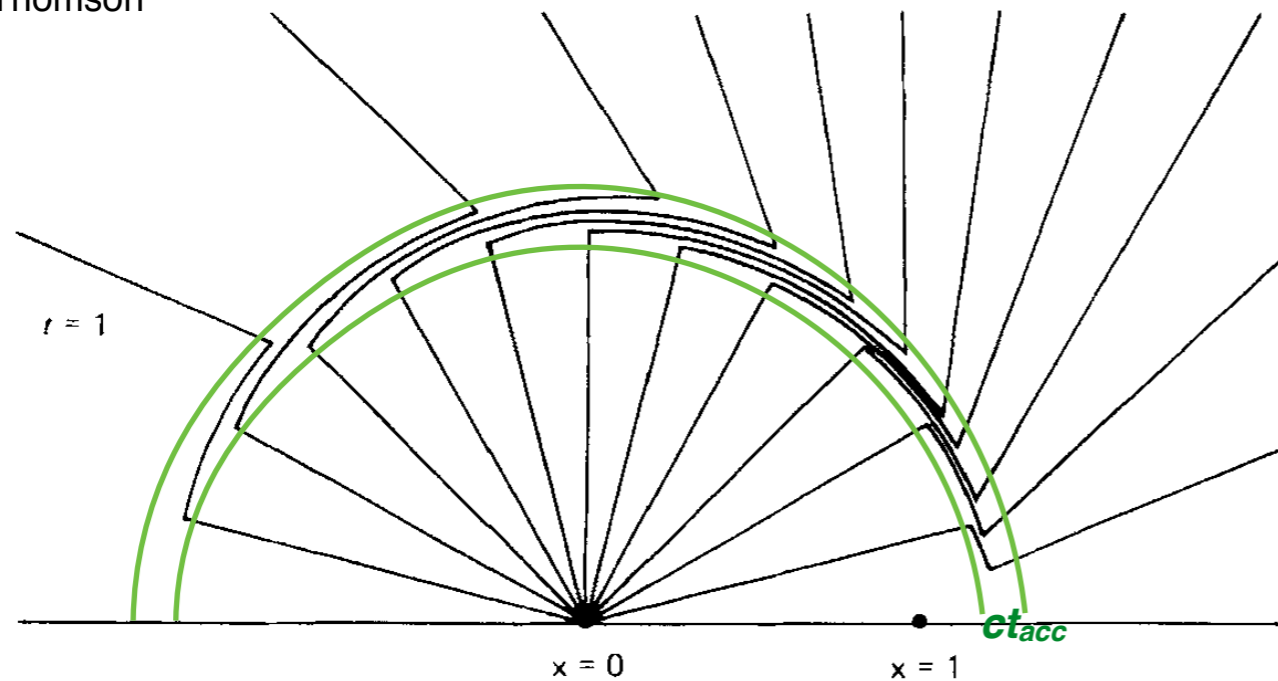


Figure 3.2 Graphical demonstration of the $1/R$ acceleration field. Charged particle moving at uniform velocity in positive x direction is stopped at $x=0$ and $t=0$.
from Rybicki & Lightman 1979

Field line density at the ring: $\sim 1/(2\pi r \cdot ct_{acc})$

Propagating transverse electric field (radiation):

$$E_{\theta} = \frac{q|\mathbf{a}|\sin\theta}{4\pi\epsilon_0c^2r}$$

Associated transverse magnetic field:

$$|\mathbf{H}| = E_{\theta}/(\mu_0/\epsilon_0)^{1/2}$$

$$\text{Emitted Power} = - \left(\frac{dE}{dt} \right) = \int |\mathbf{E} \times \mathbf{H}| r^2 d\Omega = \frac{q^2 |\mathbf{a}|^2}{6\pi \epsilon_0 c^3}$$

(Larmor Formula)

\mathbf{a} = proper acceleration in the instantaneous rest frame of the charged particle

Moving Observer: $dE/dt = dE'/dt'$ (Lorentz invariant)

proper acceleration

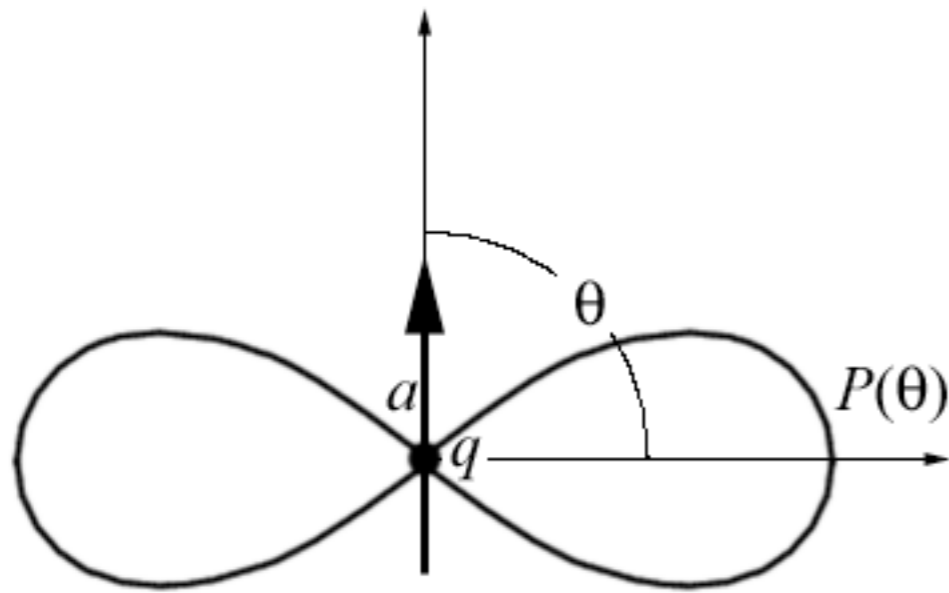
$$a_0^2 = \gamma^4 (|a_{\perp}|^2 + \gamma^2 |a_{\parallel}|^2)$$

$\gamma =$ Lorentz factor

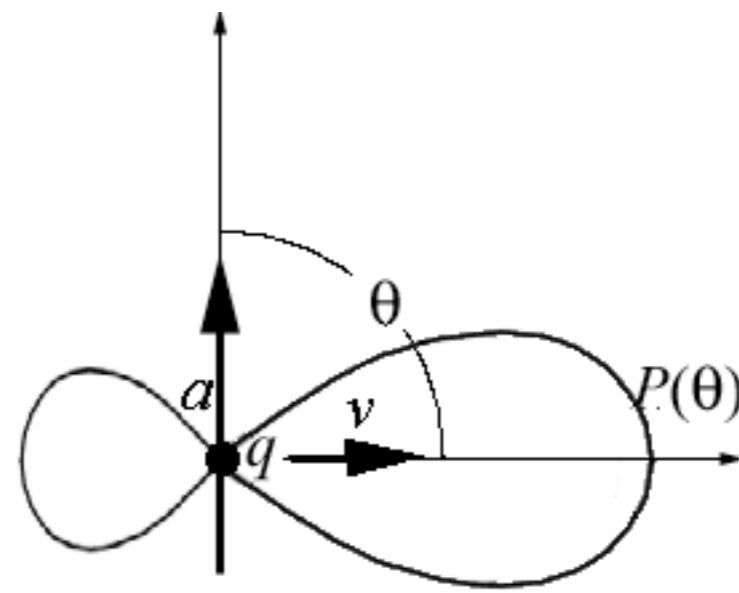
$$= \frac{1}{1 - \frac{v^2}{c^2}}$$

$$\left(\frac{dE}{dt} \right)_S = \frac{q^2 \gamma^4}{6\pi \epsilon_0 c^3} (|a_{\perp}|^2 + \gamma^2 |a_{\parallel}|^2)$$

Radiation Pattern



Stationary
dipole



Moving
dipole

Motion introduces aberration and relativistic beaming

Polarization

$$\vec{E} \propto \hat{n} \times [(\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}]$$

At a single particle level, over short times, radiation is always polarized.

For slowly moving particle (or $\vec{\beta}$ nearly \parallel to \hat{n}) polarization is \parallel to the projected instantaneous acceleration.

Net observed polarization involves average over the particle's trajectory, and over the distribution of emitting particles.

Spectrum:

Fourier Transform of the time-varying electric field

$$\begin{array}{l} \dot{\mathbf{v}}(t) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \dot{\mathbf{v}}(\omega) \exp(-i\omega t) d\omega, \\ \dot{\mathbf{v}}(\omega) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \dot{\mathbf{v}}(t) \exp(i\omega t) dt. \end{array} \quad \left| \begin{array}{l} \int_{-\infty}^{\infty} |\dot{\mathbf{v}}(\omega)|^2 d\omega = \int_{-\infty}^{\infty} |\dot{\mathbf{v}}(t)|^2 dt \\ = 2 \int_0^{\infty} |\dot{\mathbf{v}}(\omega)|^2 d\omega \end{array} \right.$$

$$\int_0^{\infty} I(\omega) d\omega = \int_0^{\infty} \frac{q^2}{6\pi\epsilon_0 c^3} 2|\dot{\mathbf{v}}(\omega)|^2 d\omega$$

$$I(\omega) = \frac{q^2}{3\pi\epsilon_0 c^3} |\dot{\mathbf{v}}(\omega)|^2 \quad \left| \begin{array}{l} \text{Specific Intensity } I_\nu \\ I_\nu / \nu^3 \quad : \quad \text{Lorentz Invariant} \end{array} \right.$$

Spectra

Radiation received from a source is the sum of emission from a large population of particles.

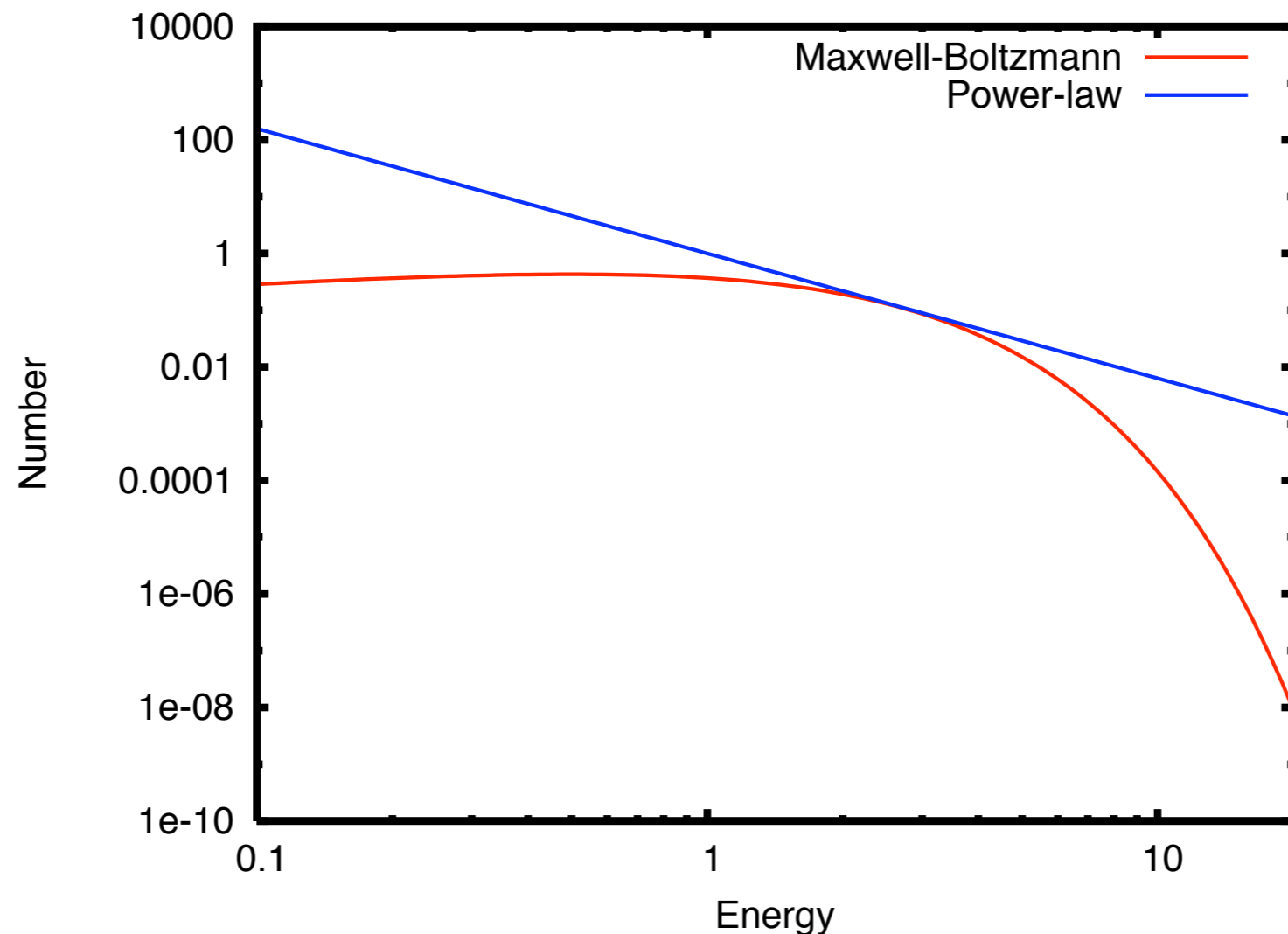
Energy distribution of the particles shape the spectra

Thermal distribution

Maxwell-Boltzmann

Non-thermal distribution

Non-Maxwellian, e.g. power-law



Radiation is modified during propagation through matter

Radiative transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

$$j_\nu = \text{emission coefficient} \quad \epsilon_\nu = \int j_\nu d\Omega$$

$$\alpha_\nu = \text{absorption coefficient}$$

$$S_\nu \equiv j_\nu / \alpha_\nu ; \quad d\tau_\nu = \alpha_\nu ds$$

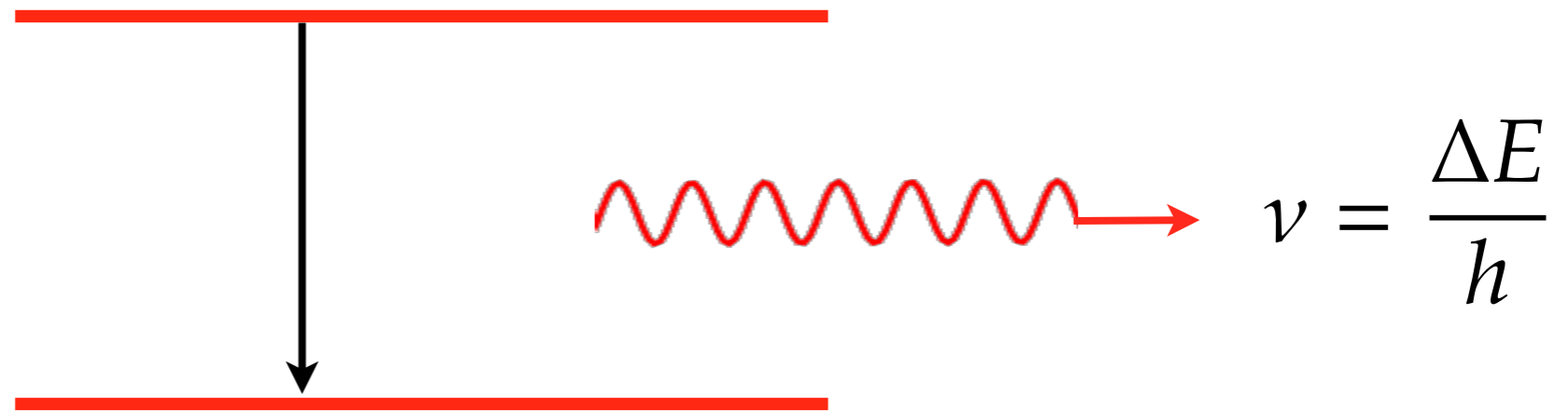
$$I_\nu = I_\nu^0 e^{-\tau_\nu} + S_\nu (1 - e^{-\tau_\nu})$$

S_ν for a thermal source is the Planck function B_ν

$$B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1}$$

Quantum Mechanical View

All e.m. radiation arises from transition between levels with difference in electric or magnetic moment



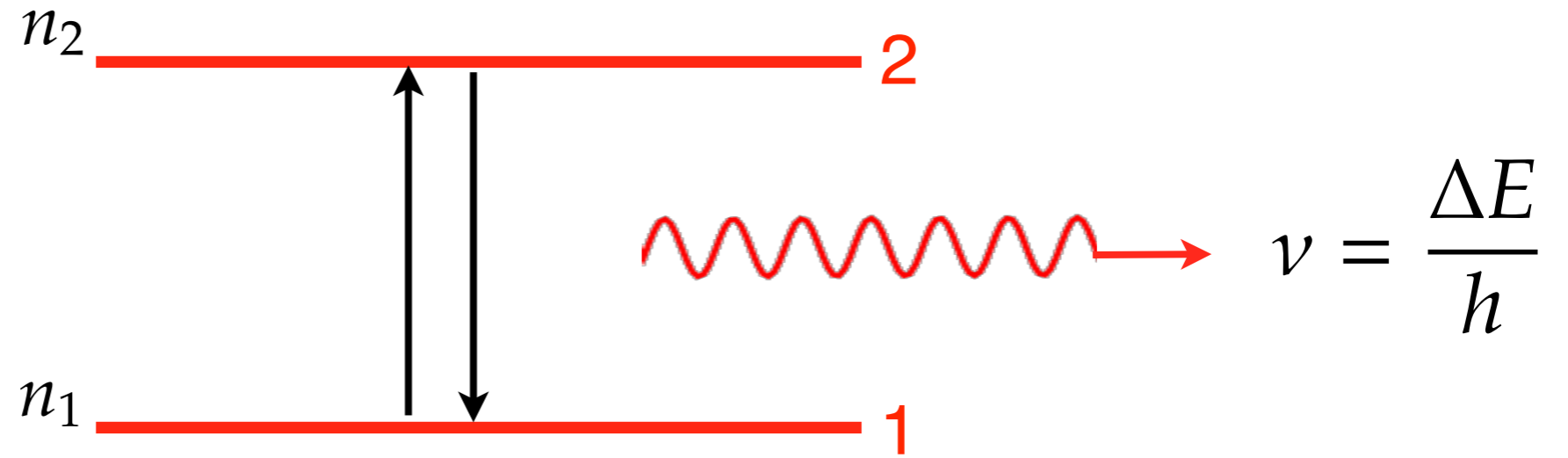
- Levels could be discrete or in continuum
- Between each pair of levels emission and absorption
- Transitions dipole / higher multipole

$$\text{Transition probability} \propto | \langle f | \exp(i\vec{k}\cdot\vec{r}) \vec{l}\cdot\sum\vec{\nabla}_j | i \rangle |^2$$

[dipole approximation $\exp(i\vec{k}\cdot\vec{r}) = 1$]

All emission processes have their inverse (absorption, stimulated emission)

$$g_2 B_{21} = g_1 B_{12} \quad ; \quad A_{21} = 2h\nu^3 B_{21} / c^2$$



$$j_\nu = \frac{h\nu}{4\pi} n_2 A_{21}$$

$$\alpha_\nu = \frac{h\nu}{4\pi} (n_1 B_{12} - n_2 B_{21})$$

$\alpha_\nu < 0$ if $\frac{n_2}{n_1} > \frac{g_2}{g_1}$: population inversion and maser emission

Blackbody constitutes the maximum emission by a thermal source at a given temperature

$$I_\nu = B_\nu(T) + e^{-\tau_\nu}(I_\nu^0 - B_\nu(T))$$

In Rayleigh-Jeans' regime $B_\nu(T) = 2\nu^2kT/c^2$

Let background temperature $T_{\text{bg}} = c^2I_\nu^0/2\nu^2k$

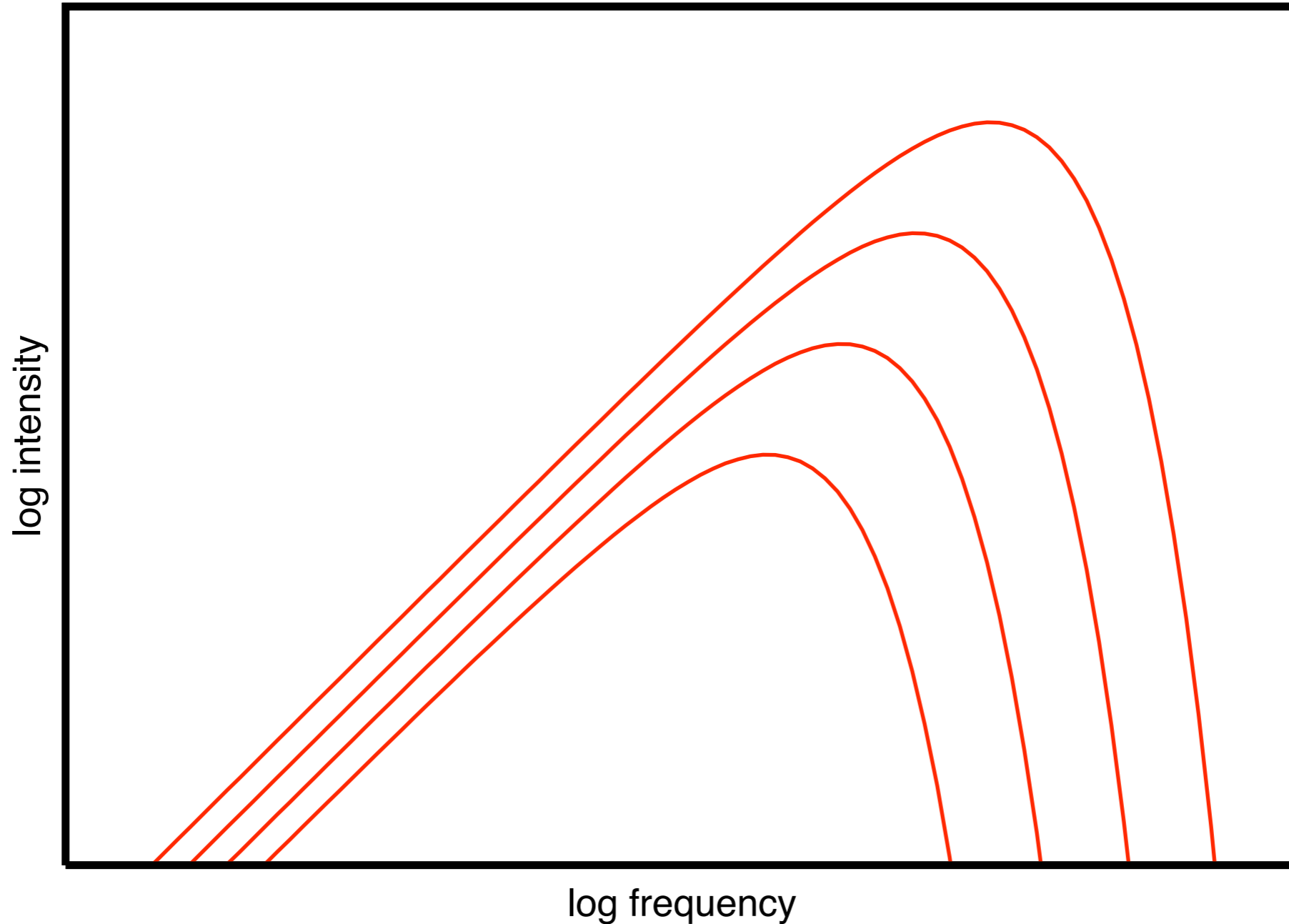
And "Brightness Temperature" $T_b = c^2I_\nu/2\nu^2k$

Then $T_b = T + e^{-\tau_\nu}(T_{\text{bg}} - T)$: lies between T_{bg} and T

$T < T_{\text{bg}}$: "absorption"; $T > T_{\text{bg}}$: "emission"

Spectral lines: τ_ν high over a narrow frequency range

Blackbody function



**At large optical depth a thermal source will emit blackbody intensity.
Emission will be received from a photosphere**

**Optical depth is frequency-dependent. A source could be optically thick
at some frequencies, optically thin at others.**

Radiation from accreting Compact Objects

Sources of Radiation

- Accretion disk, corona, winds, jets, surrounding material

Main radiation components

- Optically thick emission from disk: *Blackbody, multi-temperature*
- Spectral lines from disk photosphere: *e.g. Relativistic Fe line*
- Optically thin emission from corona, outflow, jets: *Bremsstrahlung, Synchrotron, Compton*
- Reflection from the disk: *Compton, Fluorescence*

Each has a distinct spectral signature

Matter accreting onto a compact star can have a high fraction

$\eta = GM/(Rc^2)$ of its rest energy extracted.

$\eta \sim 10\%$ for NS/BH, 0.03% for WD. (compare: $\sim 0.7\%$ for H burn)

If converted directly to thermal energy then expect (e.g. shock)

$$kT \sim \eta m_p c^2 \sim 100 \text{ MeV} \left(\frac{\eta}{0.1} \right)$$

In practice energy release is more gradual, at lower temperatures

Maximum Radiative Luminosity (Eddington Limit)

$$L_{\text{Edd}} = \frac{4\pi G \mu m_p c}{\sigma_T} M = 1.26 \times 10^{38} \text{ erg/s} \left(\frac{M}{M_\odot} \right) \mu$$

(inward gravitational force balanced by outward radiative force)

Hence Eddington accretion rate

$$\dot{M}_{\text{Edd}} = \frac{L_{\text{Edd}}}{\eta c^2} = 1.4 \times 10^{18} \text{ g/s} \left(\frac{M}{M_\odot} \right) \left(\frac{0.1}{\eta} \right) \mu$$

$\dot{M} > \dot{M}_{\text{Edd}}$ may lead to heavy mass loss / common envelope evolution

Accretion Disk

Disk forms due to angular momentum + viscosity

Keplerian thin accretion disk:

$$\Omega = \sqrt{GM/R^3} \quad v_\phi = \sqrt{GM/R}$$

$$2\pi R \Sigma(R) v_r(R) = \dot{M} \quad \text{accretion rate}$$

$$\tau(R) = R(2\pi R) \nu \Sigma (R d\Omega/dR) \quad \text{viscous torque}$$

$$\dot{M} \frac{d(R^2 \Omega)}{dR} = - \frac{d}{dR} \left[\nu \Sigma 2\pi R^3 \frac{d\Omega}{dR} \right]$$

integrating,

$$\nu \Sigma = \frac{\dot{M}}{3\pi} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right]$$

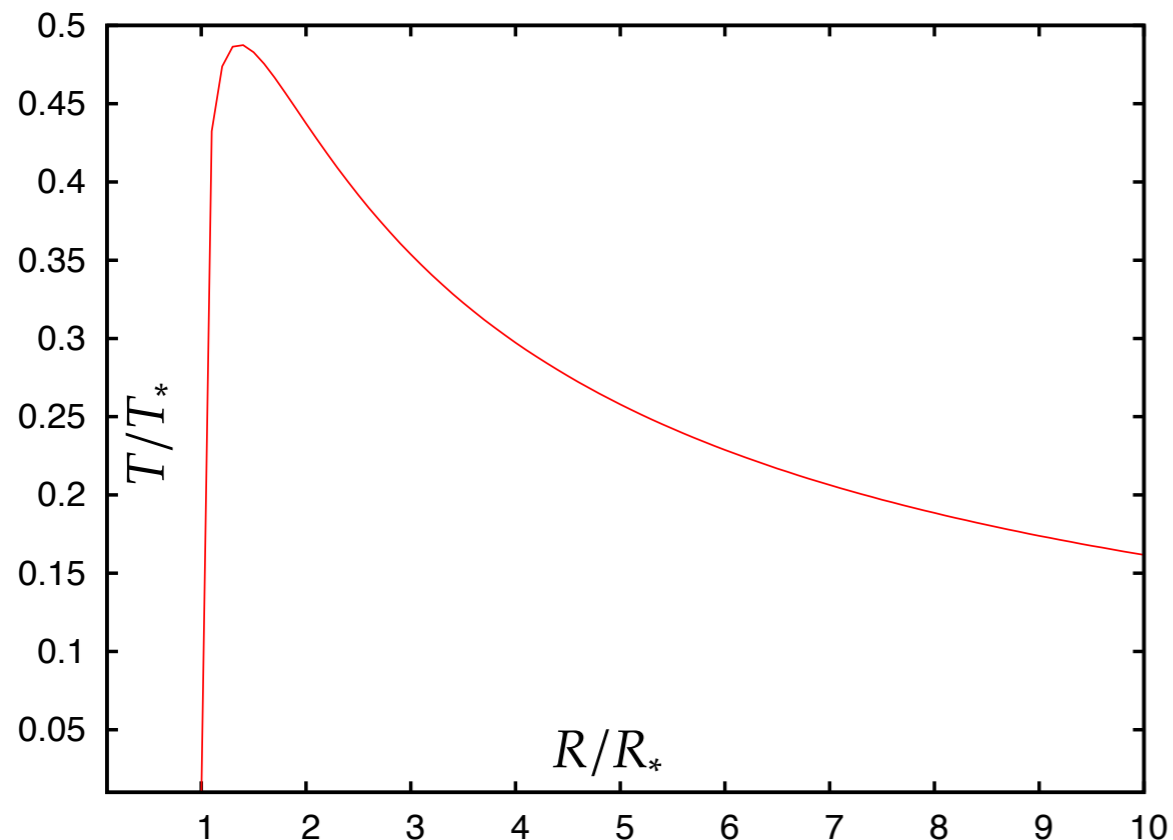
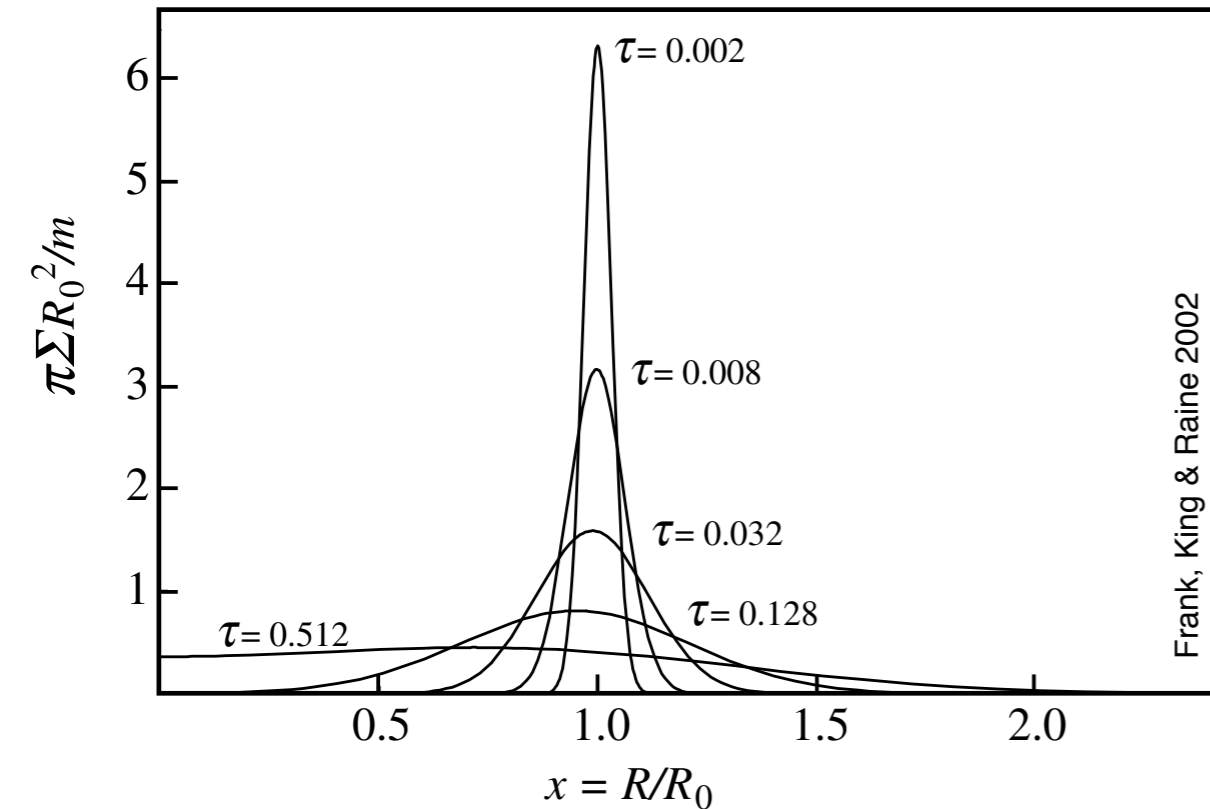
viscous dissipation rate per unit area:

$$D(R) = \nu \Sigma \left(R \frac{d\Omega}{dR} \right)^2 = \frac{3GM\dot{M}}{4\pi R^3} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right]$$

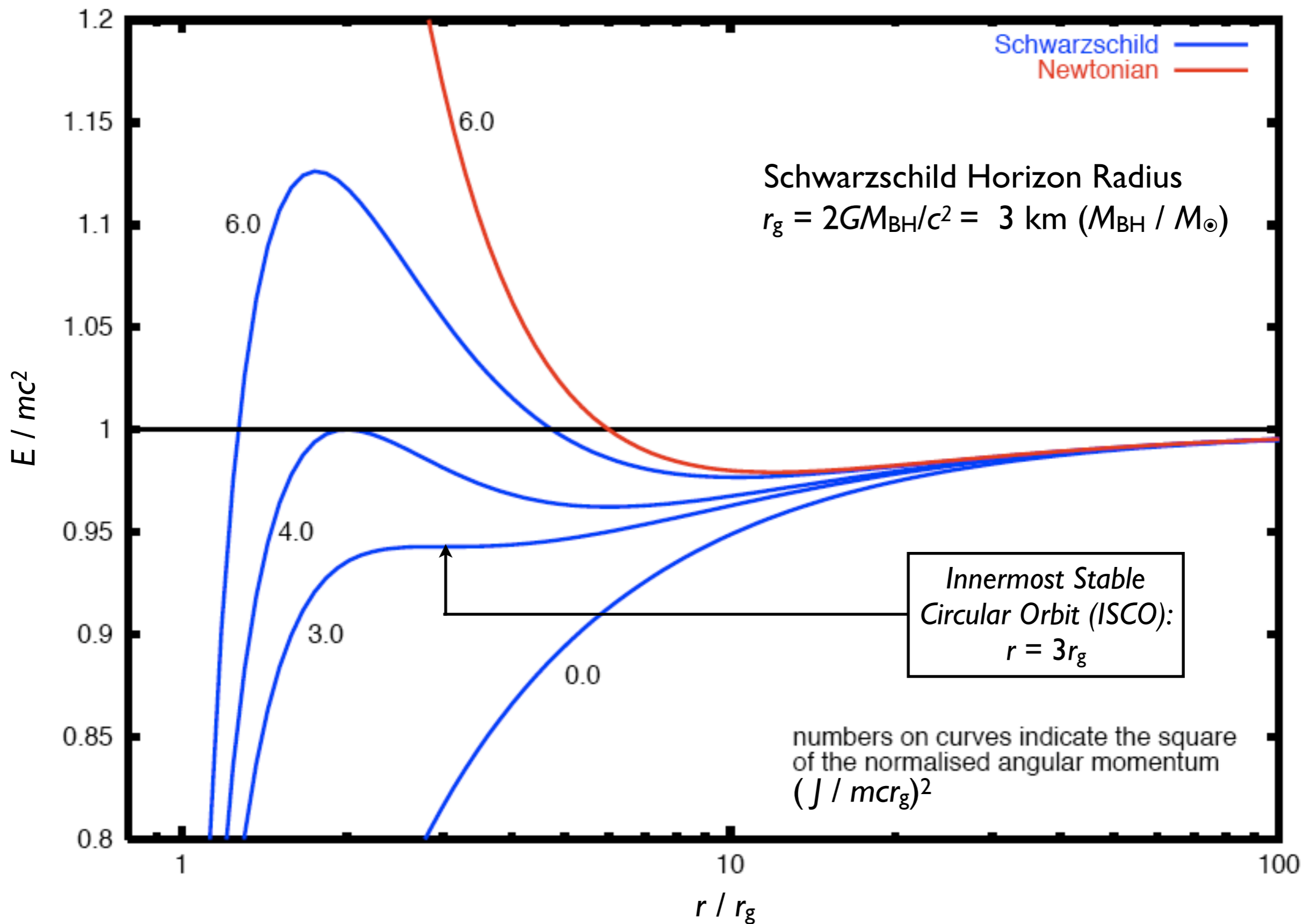
balancing this with local blackbody emission:

$$T(R) = \left\{ \frac{3GM\dot{M}}{8\pi R^3 \sigma} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right] \right\}^{1/4}$$

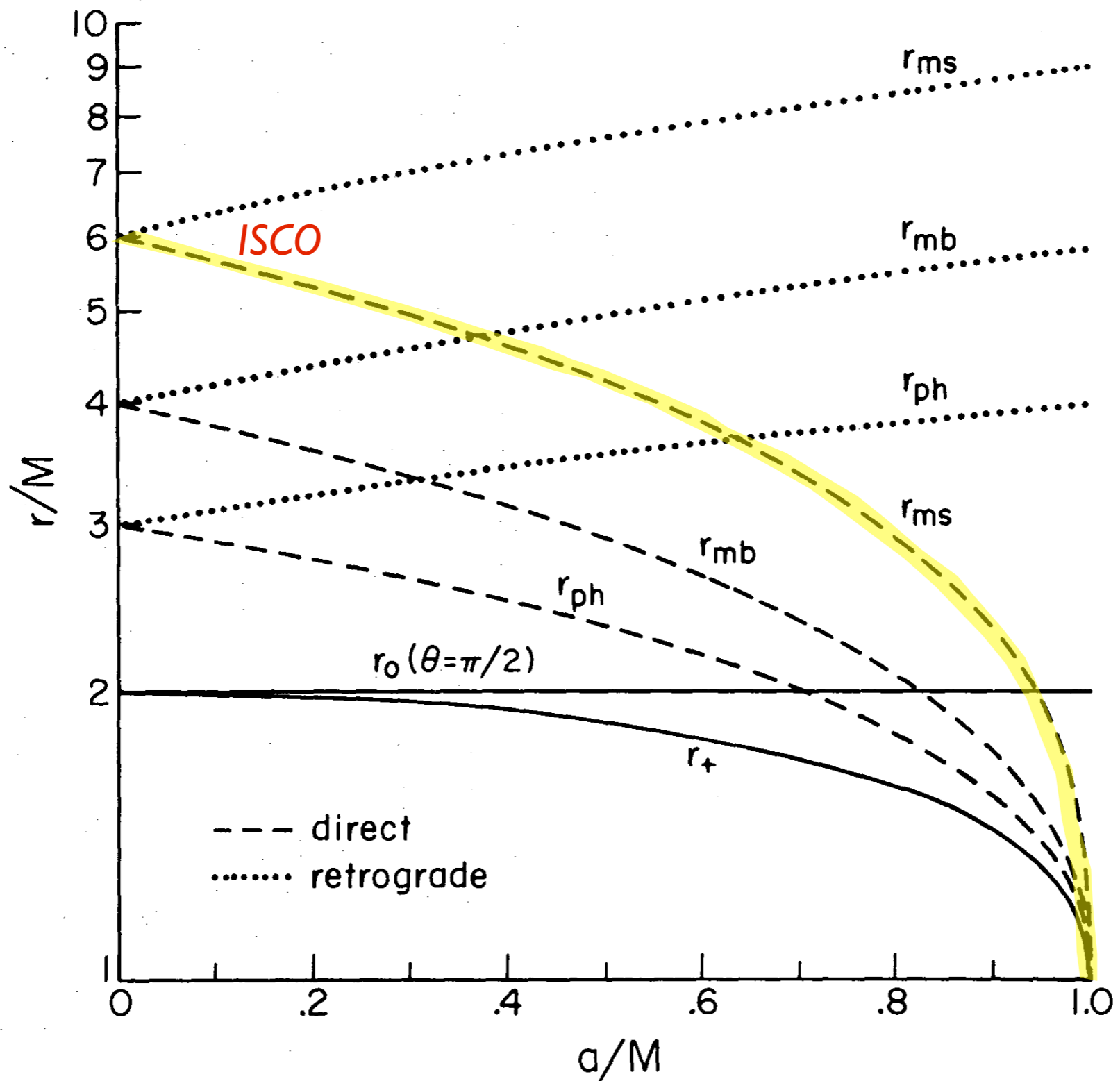
$$T_* = \left(\frac{3GM\dot{M}}{8\pi R^3 \sigma} \right)^{1/4} = 1.12 \text{ keV} (M/M_\odot)^{1/4} R_6^{-3/4} \dot{M}_{17}^{1/4}$$



Keplerian motion near a Black Hole



Critical radii in Kerr Geometry



Thermal emission from Accretion Disk

Standard Keplerian Disk can extend down to ISCO: $9(M_{\text{BH}}/M_{\odot})$ km for non-rotating Black Hole, even closer for rotating BH.

Temperature of the disk is a function of radius. For standard optically thick, geometrically thin, keplerian accretion disk

$$T(R) = T_* \left[1 - \left(\frac{R_{\text{in}}}{R} \right)^{1/2} \right]^{1/4}$$

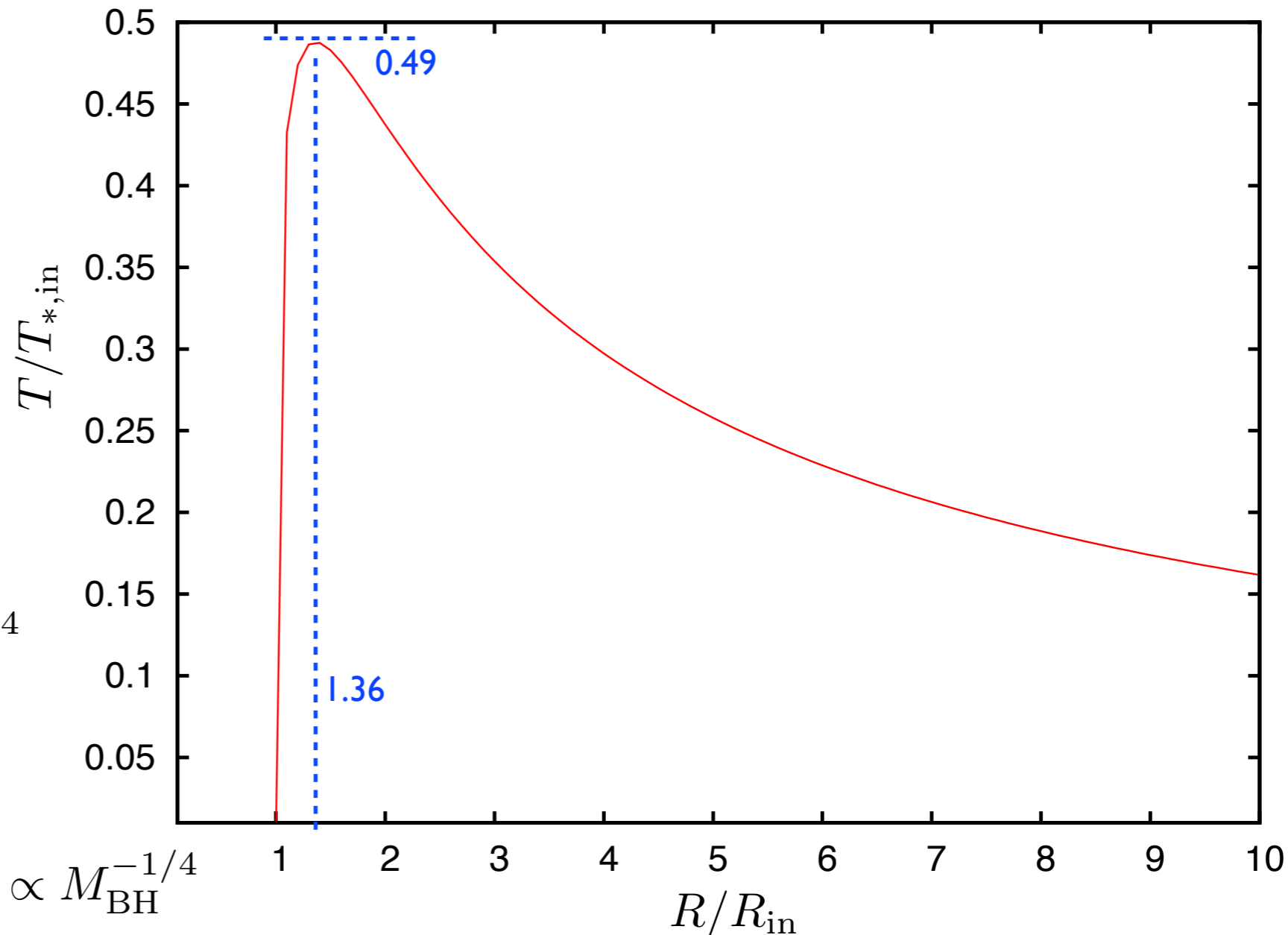
where

$$T_* = \left(\frac{3GM\dot{M}}{8\pi R^3\sigma} \right)^{1/4}$$

$$kT_* = 1.12 \text{ keV} \left(\frac{\dot{M}}{10^{17} \text{ g/s}} \right)^{1/4}$$

$$\times \left(\frac{M_{\text{BH}}}{M_{\odot}} \right)^{1/4} \left(\frac{R}{10 \text{ km}} \right)^{-3/4}$$

Typically soft emission



“Multicolour Blackbody”, $T_{\text{max}} \propto M_{\text{BH}}^{-1/4}$

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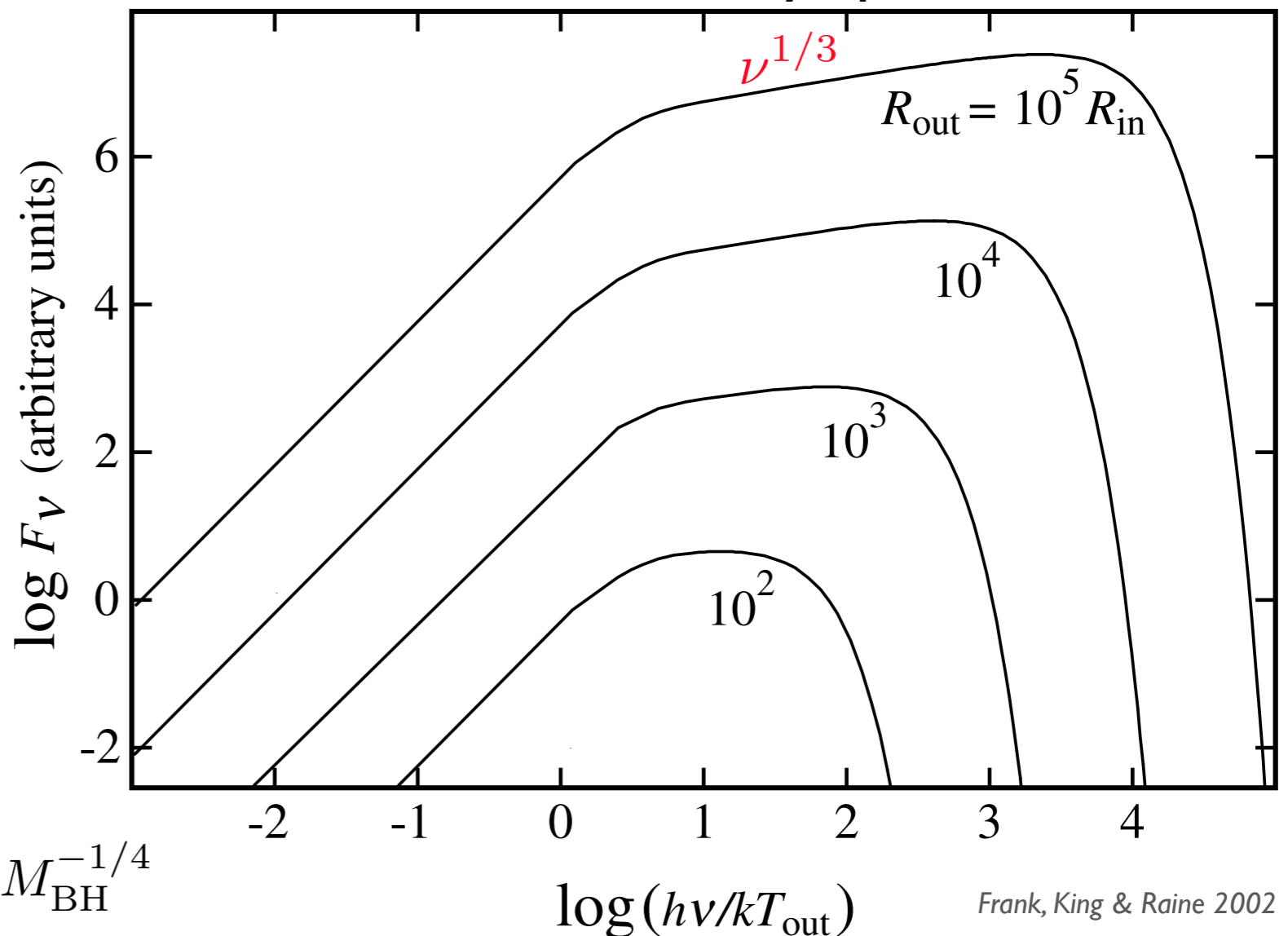
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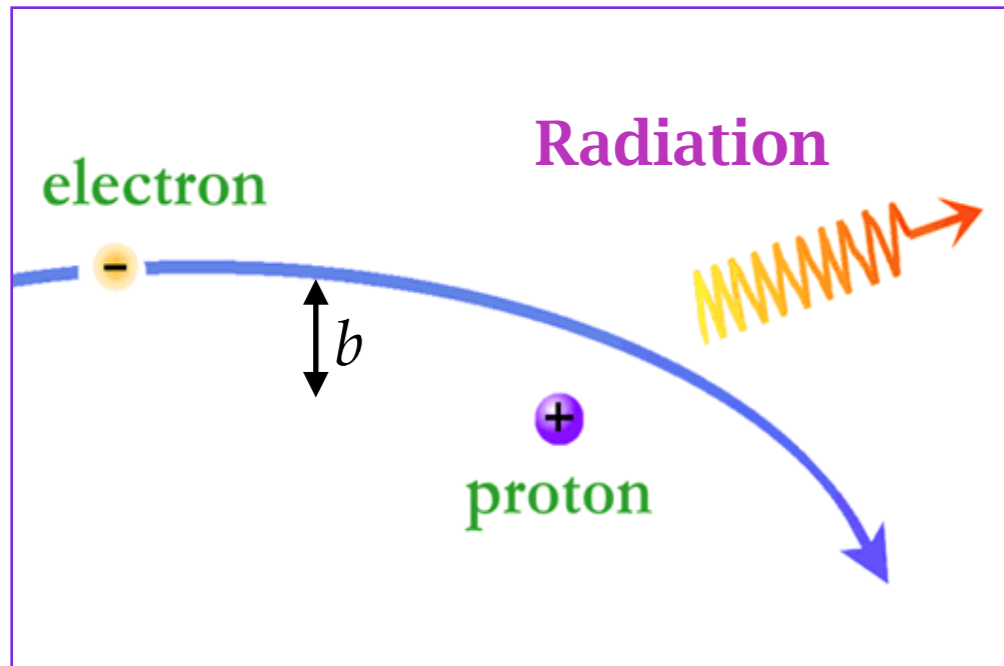
Disk Blackbody spectrum



“Multicolour Blackbody”, $T_{\text{max}} \propto M_{\text{BH}}^{-1/4}$

Continuum Emission Processes

Bremsstrahlung (*free-free emission*)



Spectrum

Electric field received by the observer is time dependent

Fourier transform of the electric field yields the spectrum

$$I(\omega) = \frac{Z^2 e^6}{24\pi^4 \epsilon_0^3 c^3 m_e^2 v^2} \frac{\omega^2}{\gamma^2 v^2} \left[\frac{1}{\gamma^2} K_0^2 \left(\frac{\omega b}{\gamma v} \right) + K_1^2 \left(\frac{\omega b}{\gamma v} \right) \right]$$

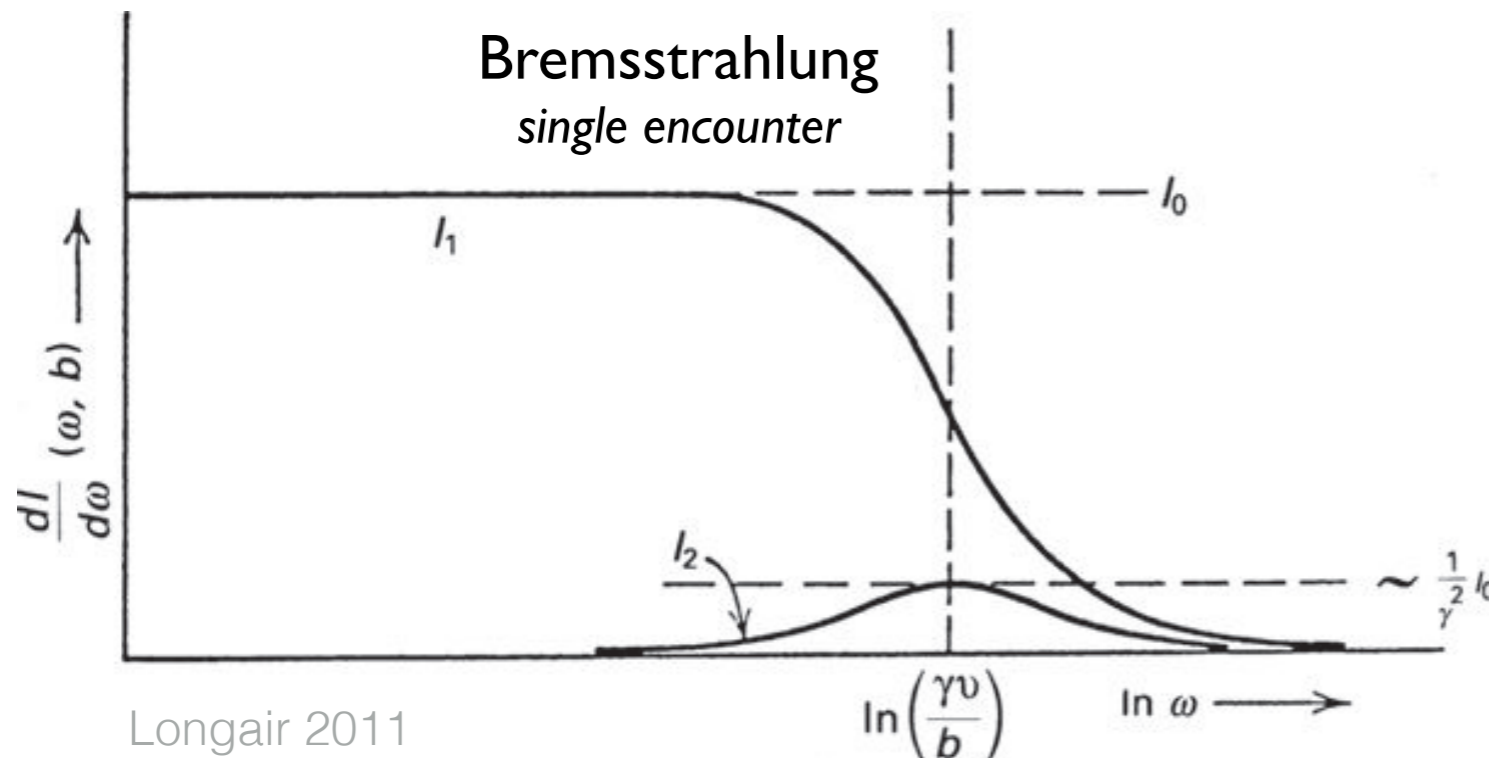
Net observed spectrum is the sum of spectra from all emitting particles

All encounters of a single electron with velocity v

$$I(\omega') = \int_{b'_{\min}}^{b'_{\max}} 2\pi b' \gamma N v K db'$$

$$= \frac{Z^2 e^6 \gamma N}{12\pi^3 \epsilon_0^3 c^3 m_e^2 v} \frac{1}{v} \ln \left(\frac{b'_{\max}}{b'_{\min}} \right)$$

Integrate this over the velocity distribution



Thermal Bremsstrahlung

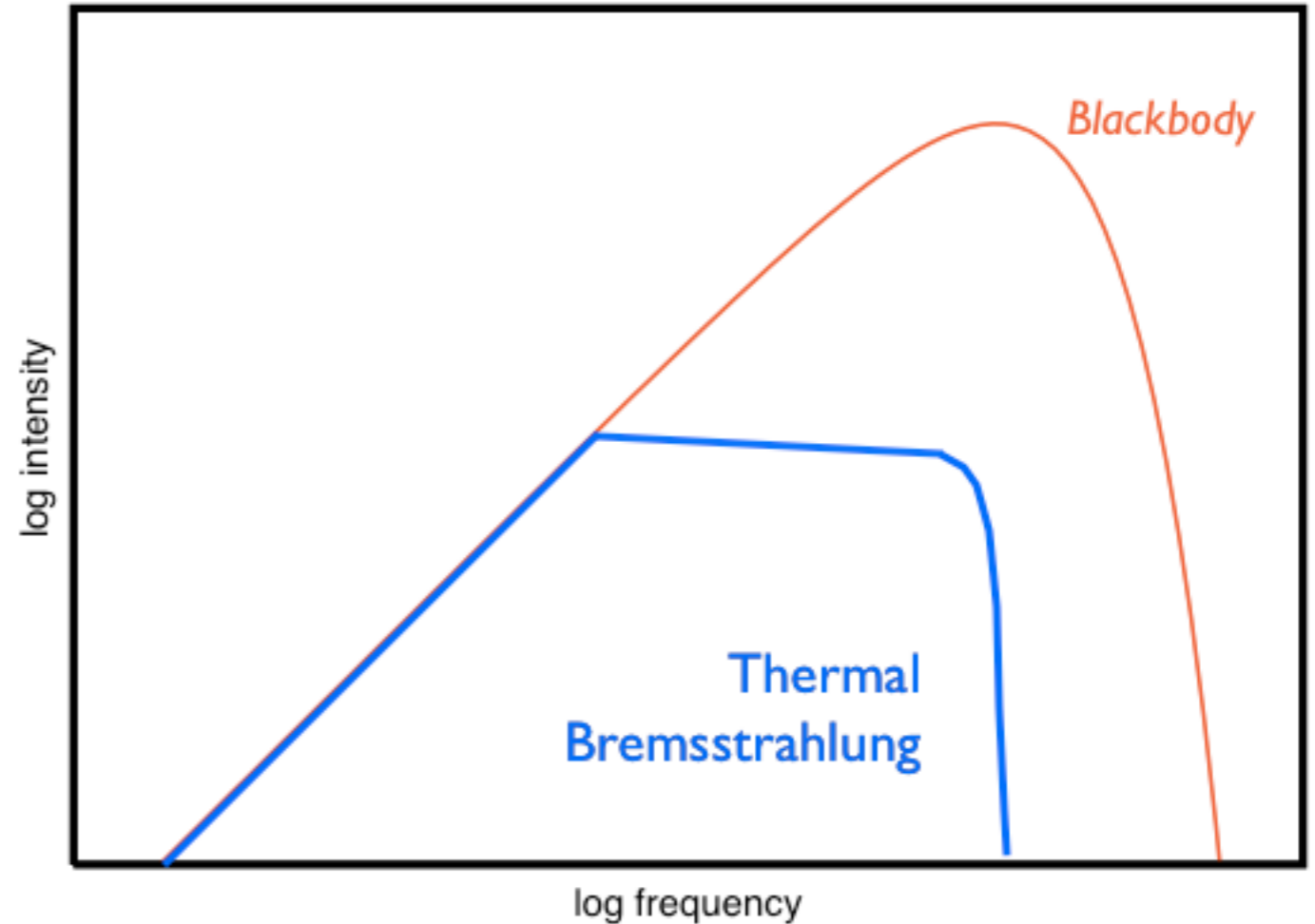
Velocity distribution (Maxwellian)

$$N_e(v) dv = 4\pi N_e \left(\frac{m_e}{2\pi kT} \right)^{3/2} v^2 \exp\left(-\frac{m_e v^2}{2kT}\right) dv$$

Emission per unit volume

$$I(\omega) \approx \frac{Z^2 e^6 N N_e}{12\sqrt{3}\pi^3 \epsilon_0^3 c^3 m_e^2} \left(\frac{m_e}{kT} \right)^{1/2} g(\omega, T)$$

$$g(\omega, T) = e^{-\hbar\omega/kT} \bar{g}_{ff}$$



Emission coefficient :

$$\epsilon_v^{ff} = 6.8 \times 10^{-38} Z^2 n_e n_i T^{-1/2} e^{-hv/kT} \bar{g}_{ff} \text{ erg s}^{-1} \text{ Hz}^{-1} \text{ cm}^{-3}$$

$$\text{Bolometric: } \epsilon^{ff} = 1.4 \times 10^{-27} Z^2 n_e n_i T^{1/2} \bar{g}_{ff} \text{ erg s}^{-1} \text{ cm}^{-3}$$

Free-free absorption coefficient:

$$\alpha_v^{ff} = 3.7 \times 10^8 Z^2 n_e n_i T^{-1/2} \nu^{-3} \left(1 - e^{-hv/kT}\right) \bar{g}_{ff} \text{ cm}^{-1}$$

$$\approx 4.5 \times 10^{-25} Z^2 n_e n_i (kT)_{\text{keV}}^{-3/2} \nu_{\text{MHz}}^{-2} \bar{g}_{ff} \text{ cm}^{-1}$$

all n values
in cm^{-3}

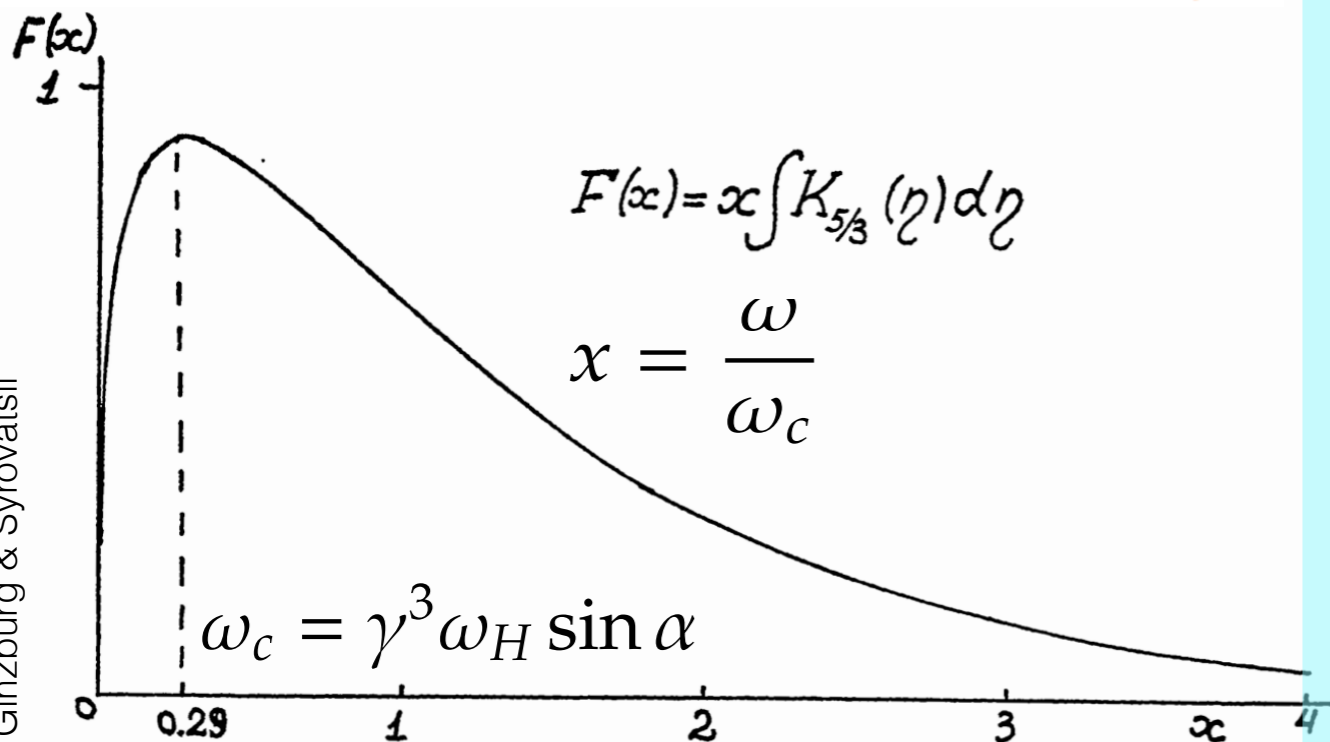
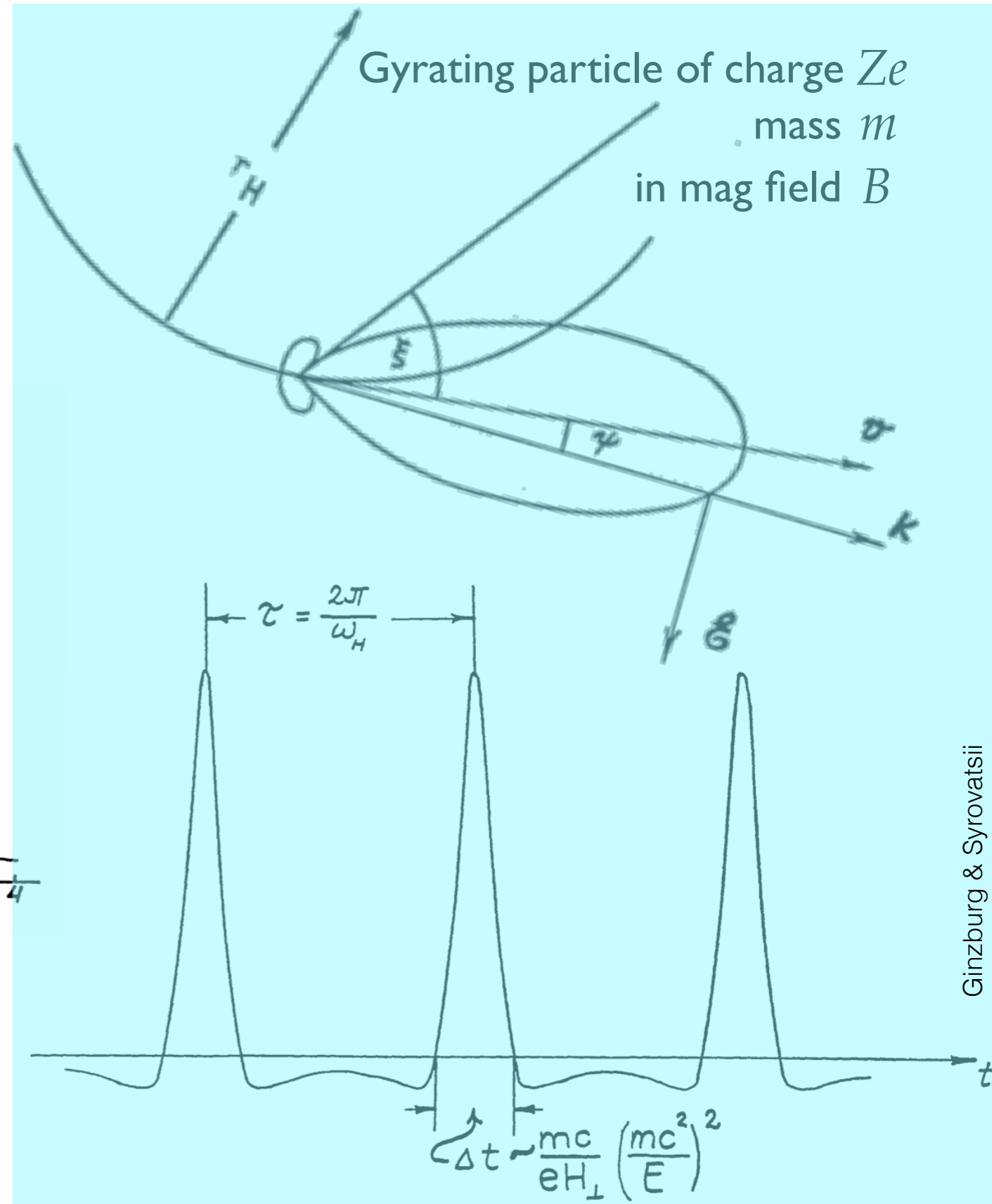
Compare Thomson:

$$\alpha_T = n_e \sigma_T$$

$$= 6.65 \times 10^{-25} n_e \text{ cm}^{-1}$$

Relativistic electron in a magnetic field

Synchrotron Radiation



$$\nu_{\text{peak}} = 0.29 \frac{\omega_c}{2\pi} \approx 0.81 \gamma^2 \left(\frac{B}{1 \text{ G}}\right) \frac{Z m_e}{m} \text{ MHz}$$

$$\text{Power} = \frac{4}{3} \sigma_{TC} \beta^2 \gamma^2 U_B \left(\frac{Z^2 m_e}{m}\right)^2$$

$$= 2.0 \times 10^{-14} \gamma^2 \left(\frac{B}{1 \text{ G}}\right)^2 \left(\frac{Z^2 m_e}{m}\right)^2 \text{ erg / s per particle}$$

Ginzburg & Syrovatskii

Ginzburg & Syrovatskii

Curvature Radiation

*Relativistic Charged Particles moving **along** curved field lines*

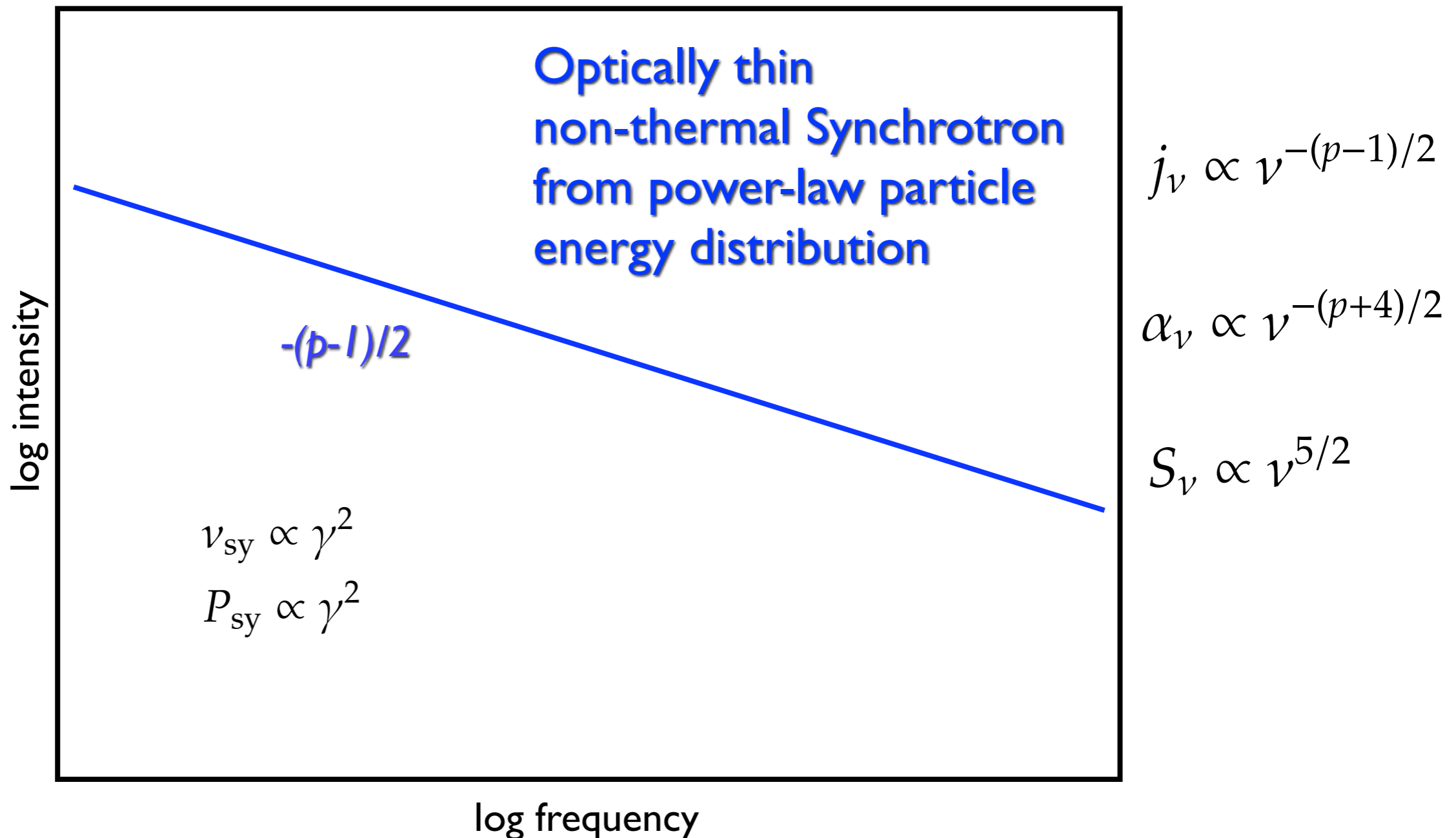
- Shares most properties of Synchrotron Radiation
(replace Larmor radius by the radius of curvature of field lines)
- Polarization || to the projected field lines
(Synchrotron: polarization perp. to projected B)

Important in pulsar magnetospheres, magnetars

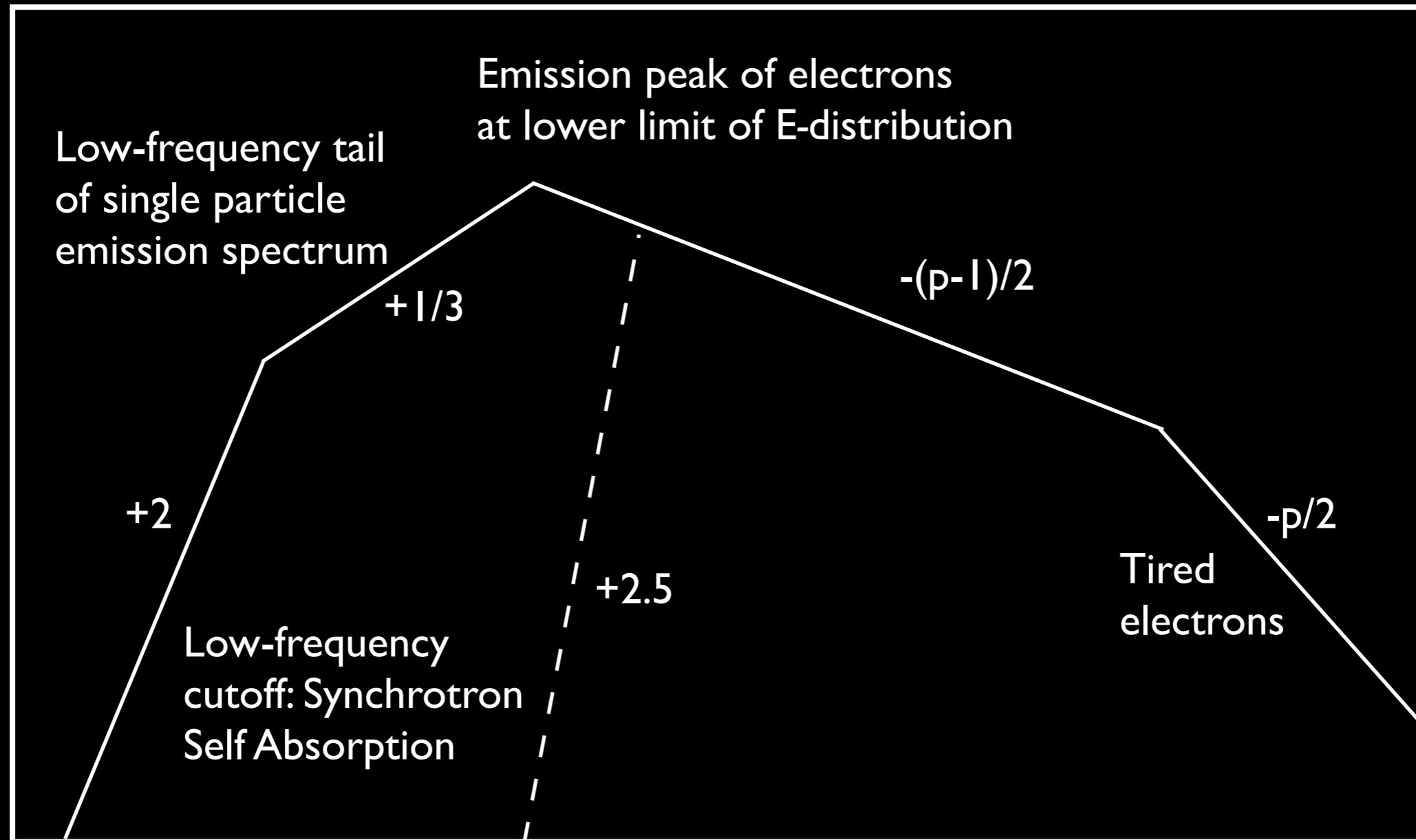
Non-thermal Emission

Acceleration processes generating relativistic particles often produce a non-thermal, power-law distribution of particle energies: $N(\gamma) \propto \gamma^{-p}$

This produces a power-law synchrotron radiation spectrum:



Spectral regimes in Synchrotron Emission



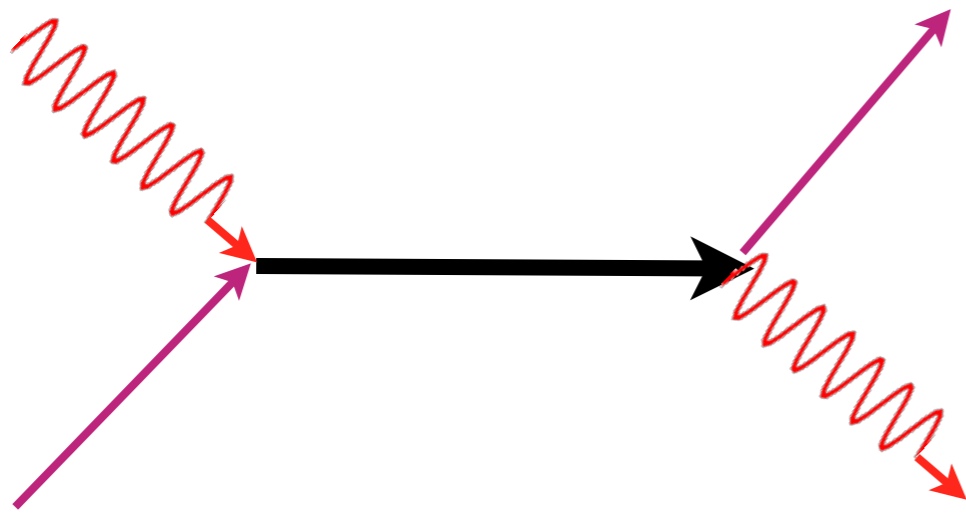
Jitter radiation can steepen the low-frequency cutoff:

- Low energy particles have longer duration of E-field pulse per orbit
- More affected by pitch angle scattering before pulse completion

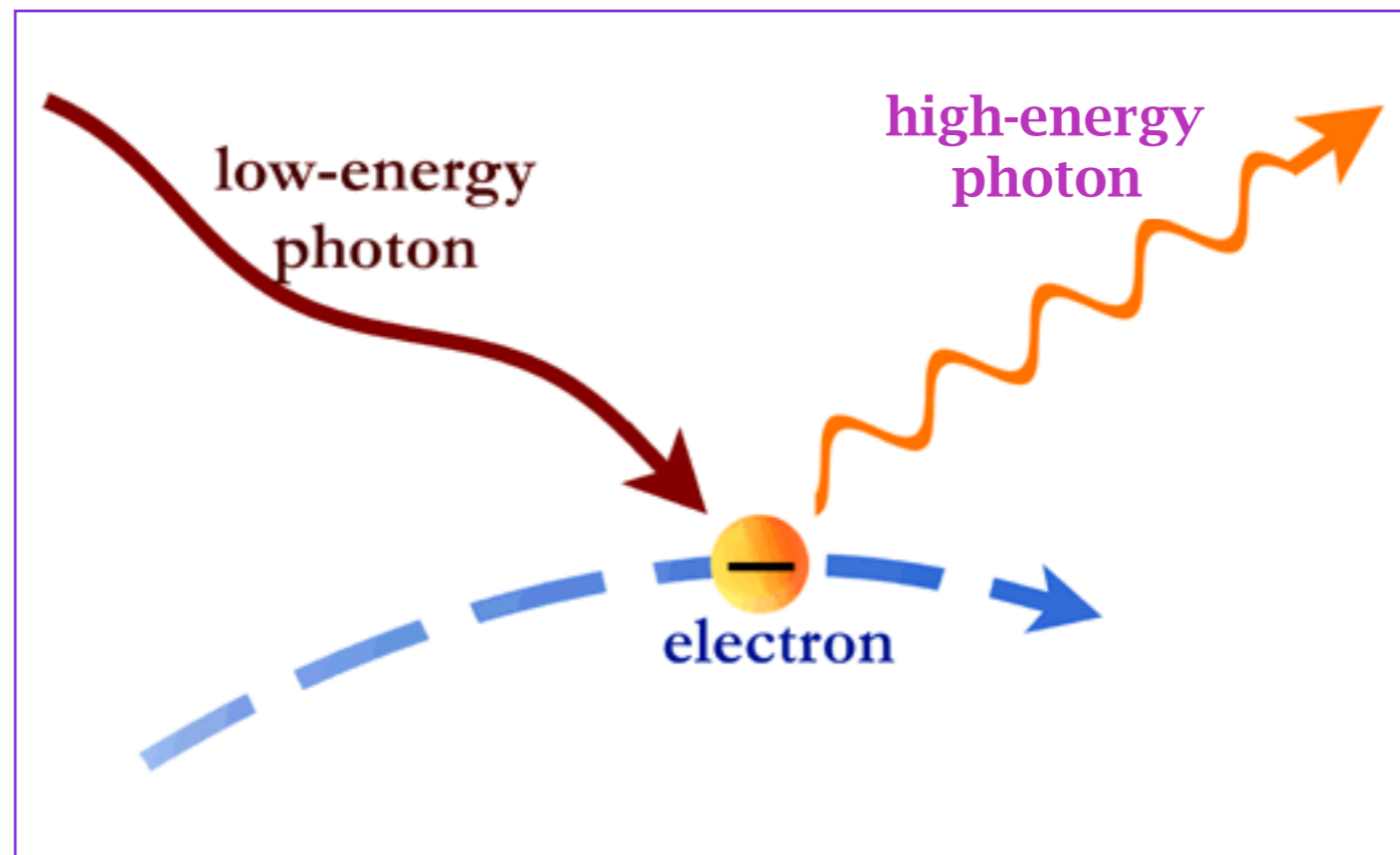
(Medvedev 2000)

Scattering processes

Non-resonant / resonant



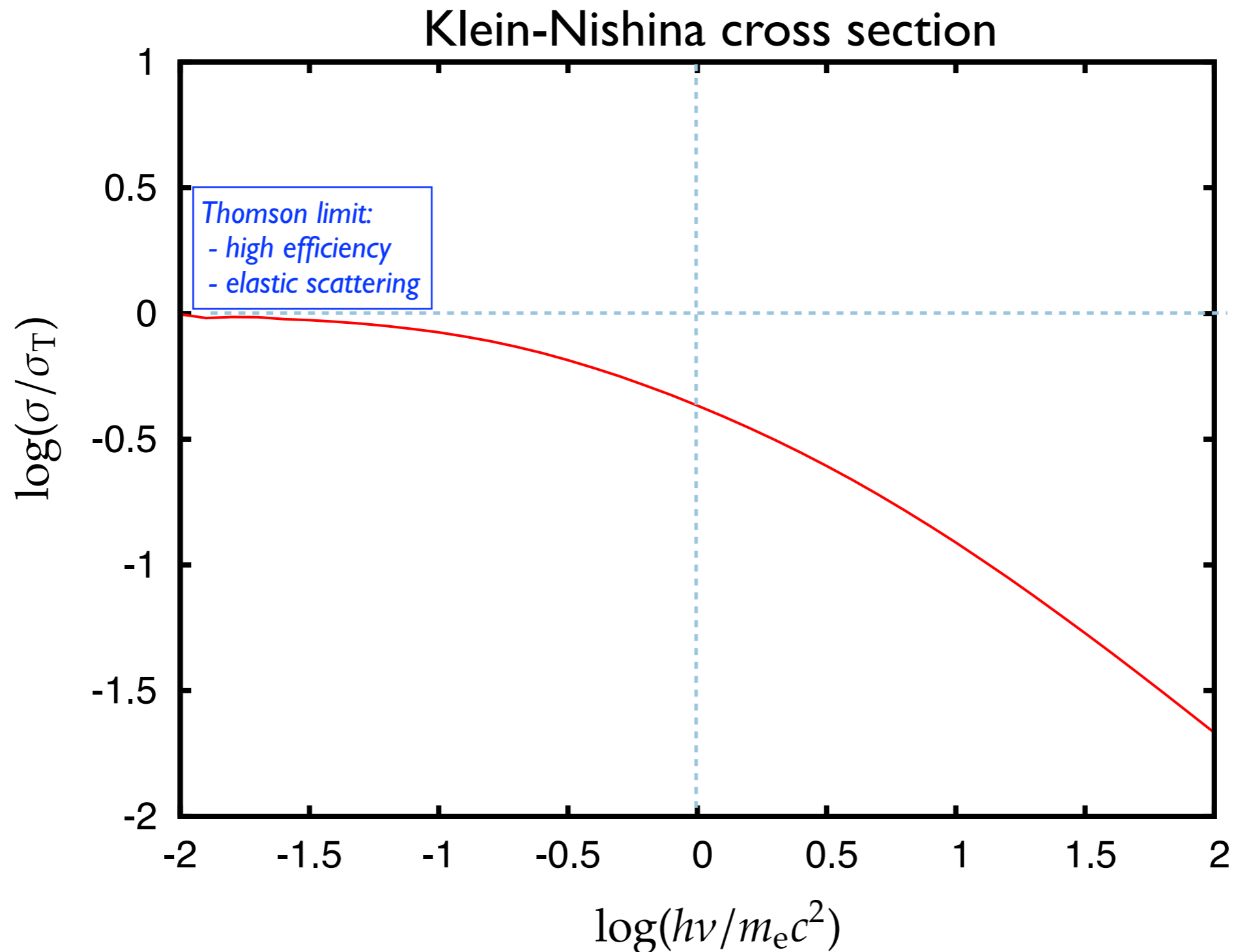
Inverse Compton Scattering



Related processes: Compton Scattering
Thomson Scattering

Compton Scattering

Thomson cross section $\sigma_T = \frac{8\pi}{3} \left(\frac{e^2}{m_e c^2} \right)^2 = 6.65 \times 10^{-25} \text{ cm}^2$



Inverse Compton Process

Synchrotron power: $\frac{4}{3}\sigma_T c \beta^2 \gamma^2 U_B$

Compton power: $\frac{4}{3}\sigma_T c \beta^2 \gamma^2 U_{\text{ph}}$
per electron

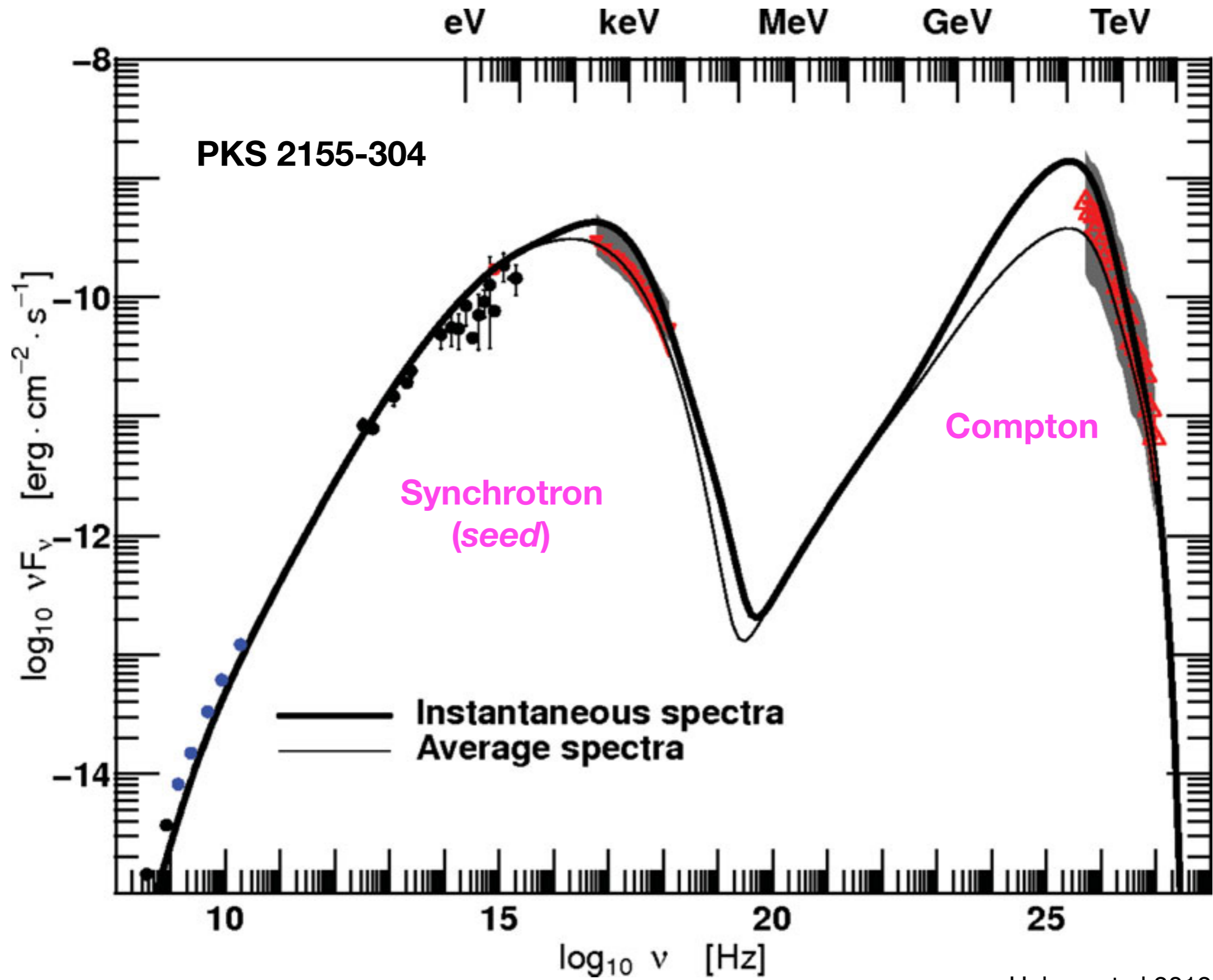
Scattered photon energy

$$\nu_i : \nu' : \nu_f = 1 : \gamma : \gamma^2$$

Like Synchrotron, here too $\nu_{\text{sc}} \propto \gamma^2$ and $P_{\text{sc}} \propto \gamma^2$

So **non-thermal** IC by electron energy distribution $N(\gamma) \propto \gamma^{-p}$
leads to a optically thin radiation spectrum $I_\nu \propto \nu^{-(p-1)/2}$

Synchrotron photons produced in an emission volume may be
compton upscattered by the same electrons: **Synchrotron Self Compton**



Thermal Comptonization

- Thermal distribution of electron energies
- Repeated scatterings
- Energy transfer both ways: electron \rightleftharpoons photon

In general, numerical radiative transfer required to describe spectra adequately

The Compton y parameter : *a measure of importance of comptonization*

$y = (\text{av. no. of scatterings}) \times (\text{mean fractional energy change per scattering})$

$$\begin{array}{l} \text{Electron scattering optical depth } \tau_{\text{es}} \sim \sigma_{\text{T}} R \\ \Rightarrow \text{Av. number of scatterings} = \text{Max}(\tau_{\text{es}}, \tau_{\text{es}}^2) \end{array} \quad \left| \begin{array}{l} \left(\frac{\Delta\epsilon}{\epsilon} \right) = \text{Max}(\alpha, \alpha^2) \\ \text{per scattering} \end{array} \right. \quad [\alpha = 4kT/m_e c^2]$$

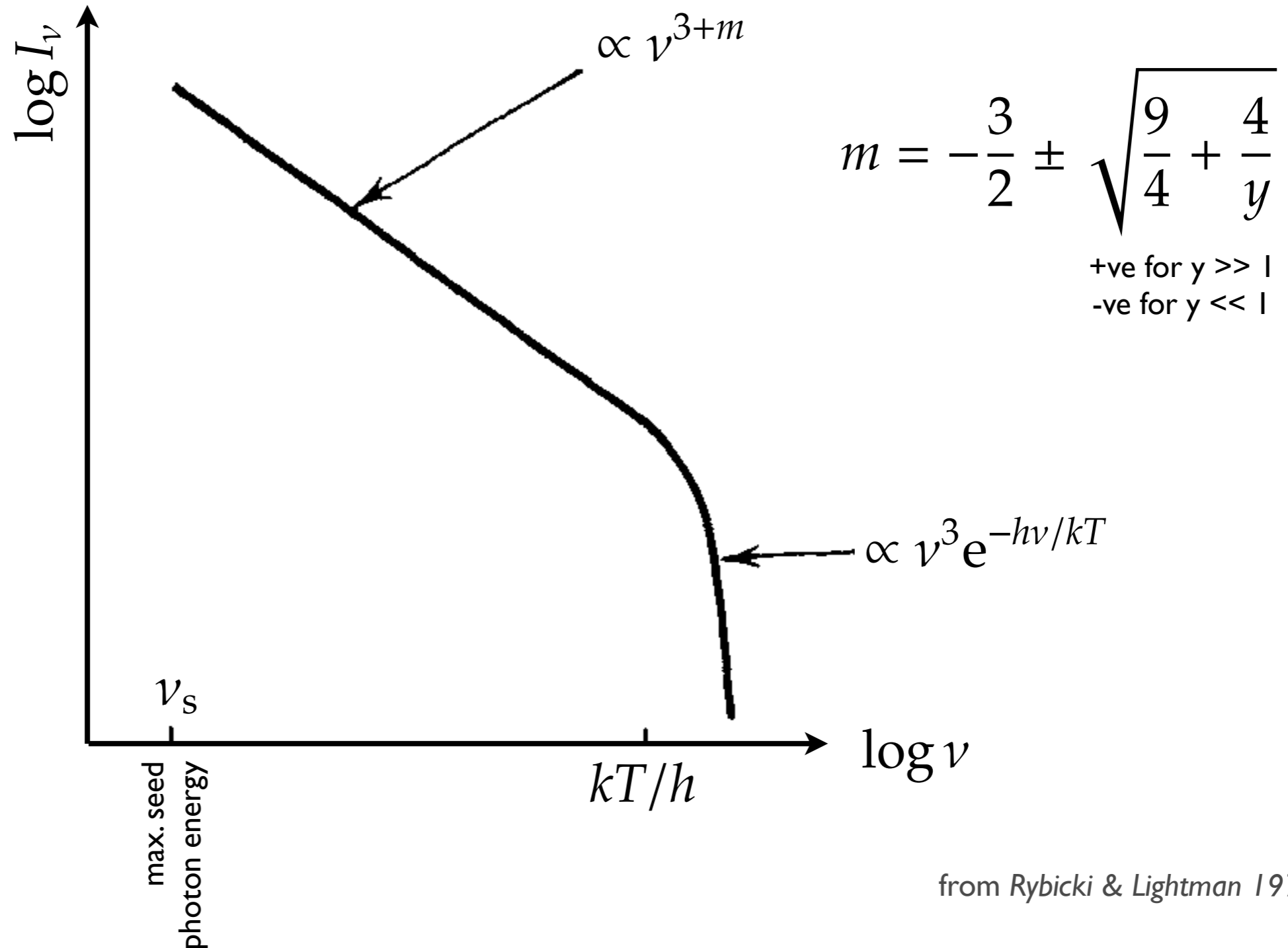
For $\alpha \gg 1$ and soft photon input ($\epsilon_i \ll m_e c^2 / \alpha$), emergent intensity $I(\epsilon) \propto \epsilon^{-k}$
if τ_{es} is small. Here $k = -\ln(\tau_{\text{es}}) / \ln(\alpha^2)$

For $\alpha \ll 1$ (non-relativistic electrons), photons random walk in energy space.

Can be described by a Boltzmann Equation \Rightarrow *Kompaneets Equation*

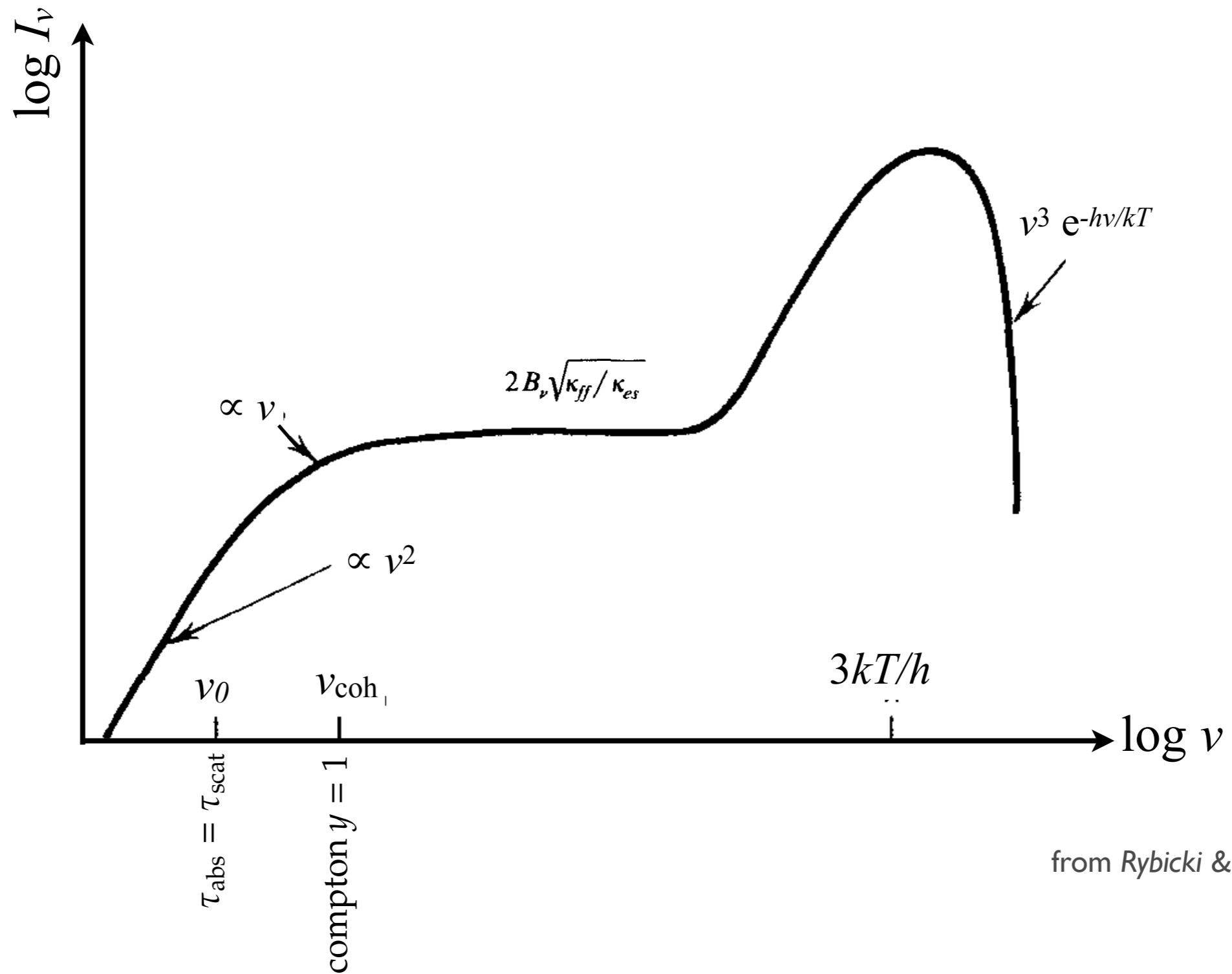
Result can be diverse, but some limiting cases have interesting properties

Non-relativistic thermal Comptonization: *Unsaturated*



from Rybicki & Lightman 1979

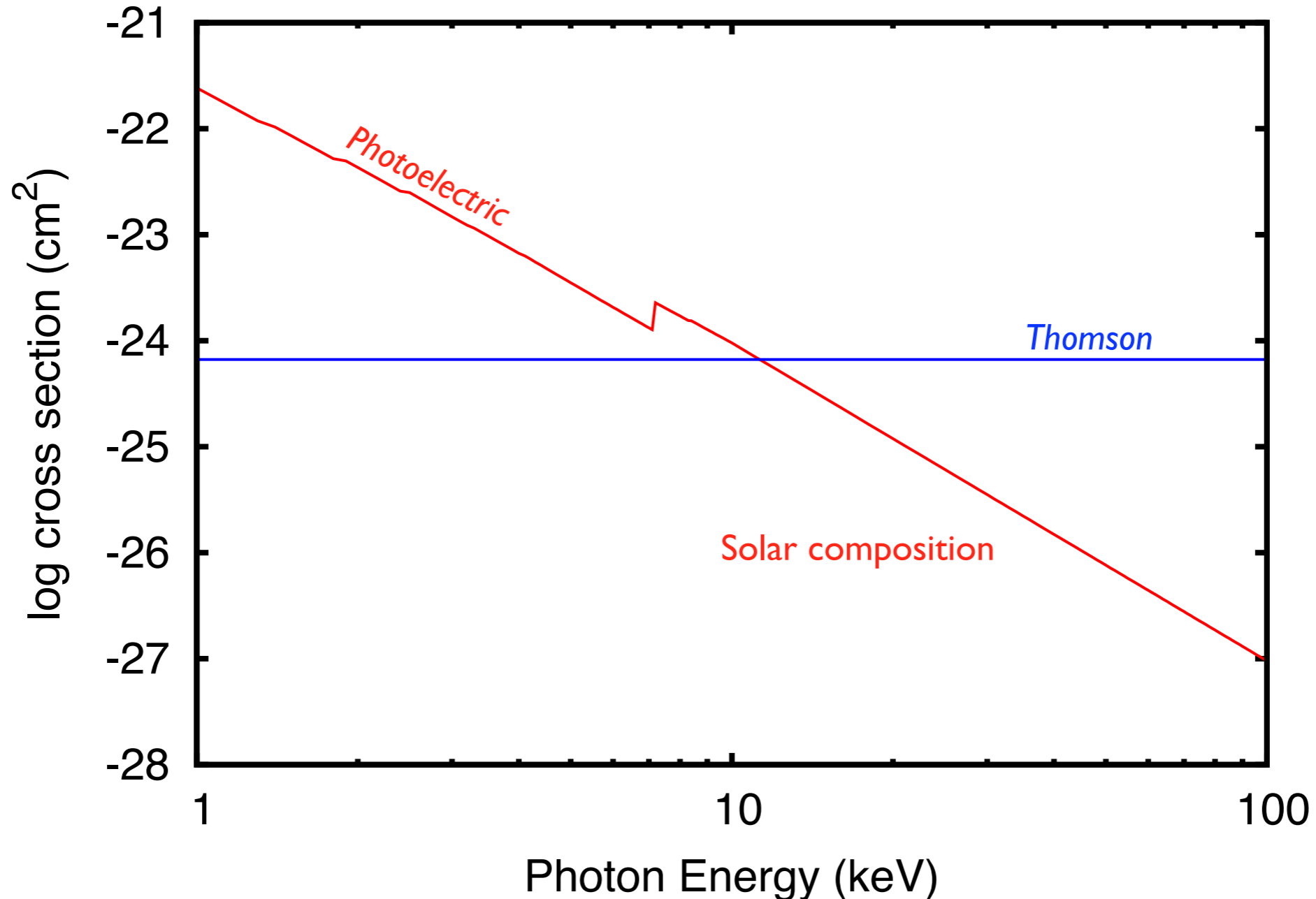
Non-relativistic thermal Comptonization: *Saturated*



from Rybicki & Lightman 1979

Compton Reflection (backscatter) from the accretion disk

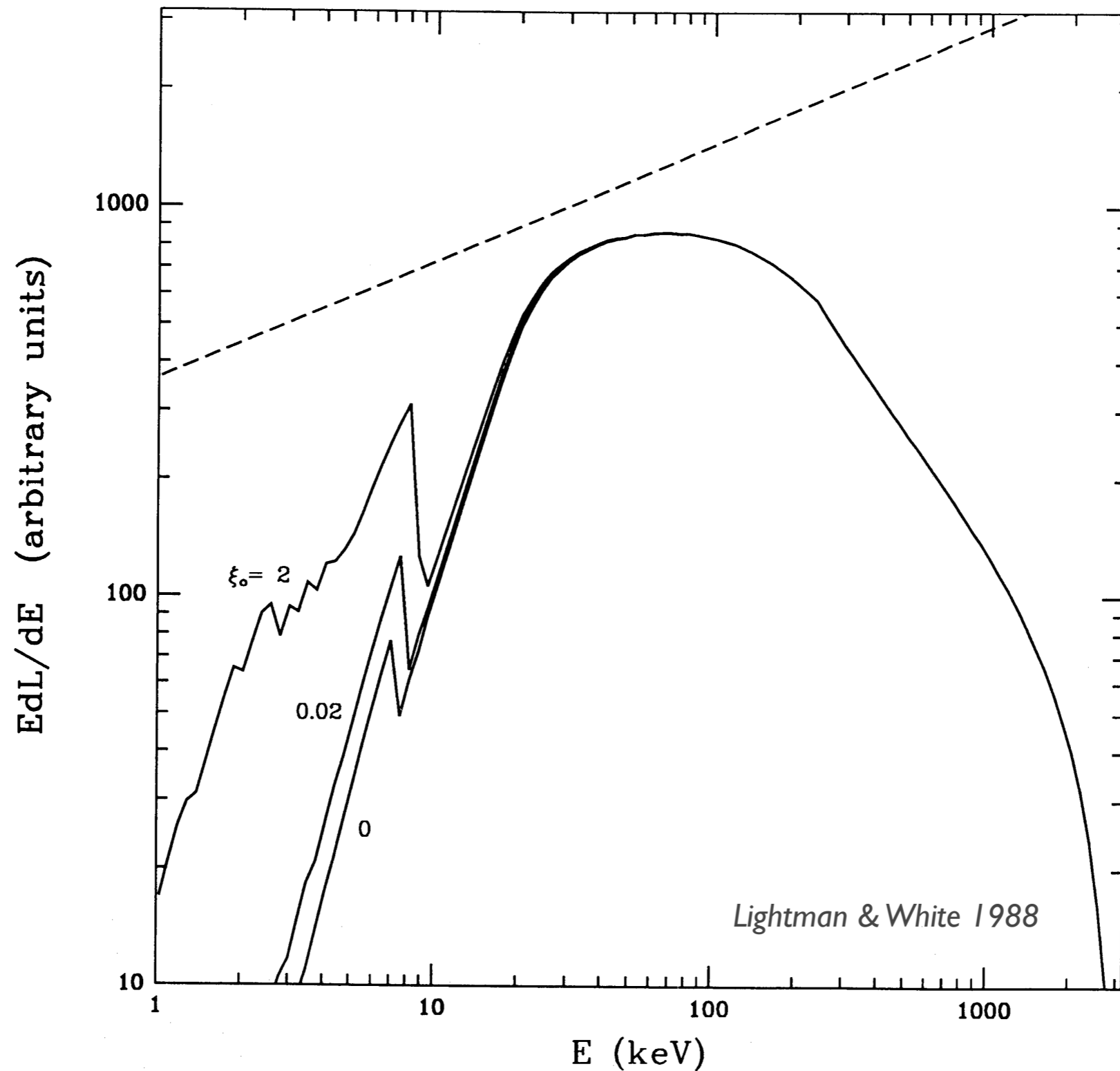
- Ineffective below 10 keV due to photoelectric absorption



- Drops above ~50 keV due to photon energy loss and KN cutoff

Results in a broad reflection hump in hard X-rays

Compton Reflection (backscatter) from the accretion disk



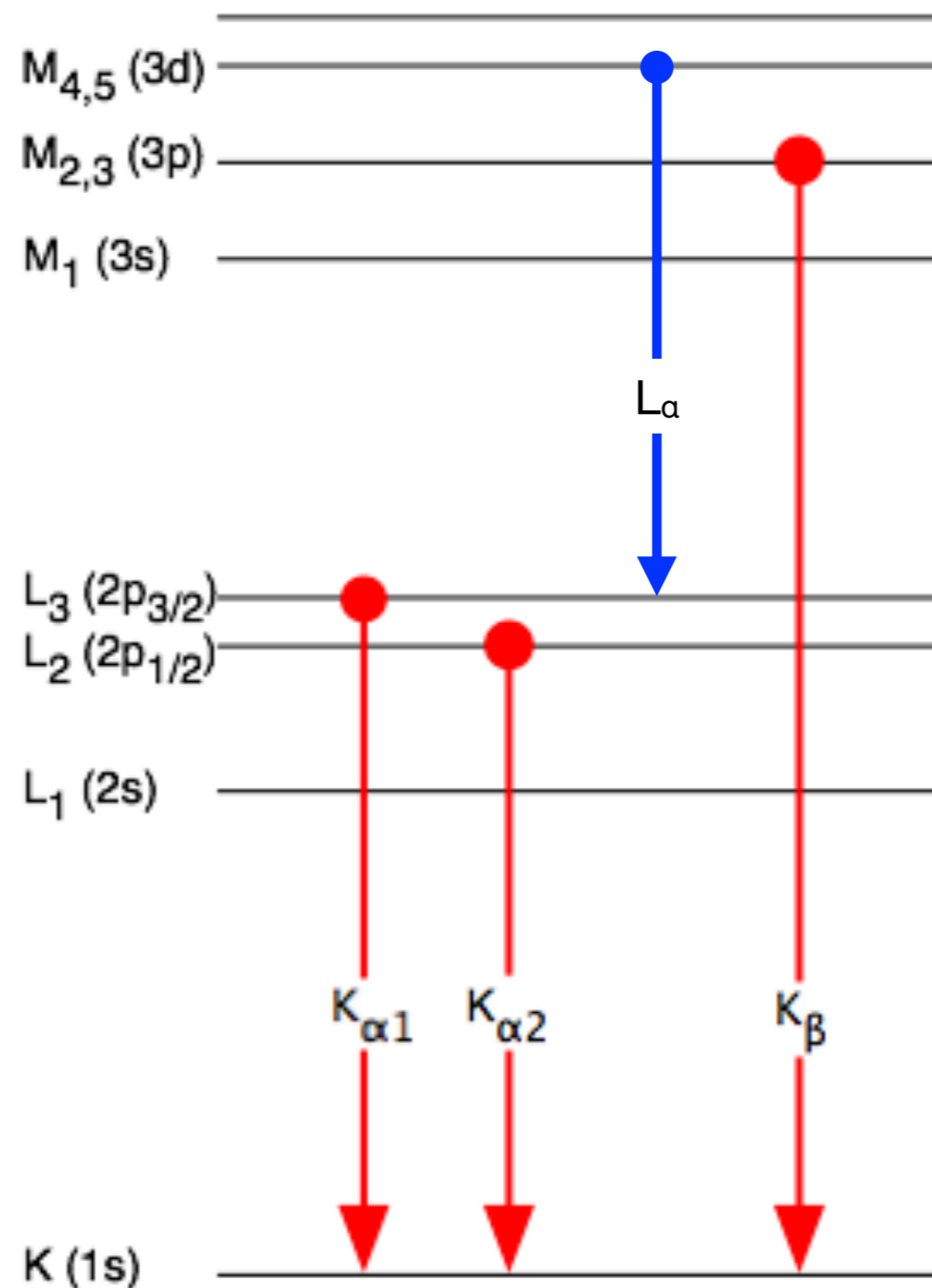
Results in a broad reflection hump in hard X-rays

Line emission

X-ray Spectral lines

Characteristic X-ray lines are generated by inner shell transitions

Can be used to identify elements



X-ray Spectral lines

Gas at high temperature may be highly ionised
Abundance of different ionisation states determined
by statistical equilibrium

- Photoionisation
- Collisional ionisation
(*impact, charge exchange*)
- Autoionisation
- Radiative recombination
- Charge exchange
- Dielectronic recombination

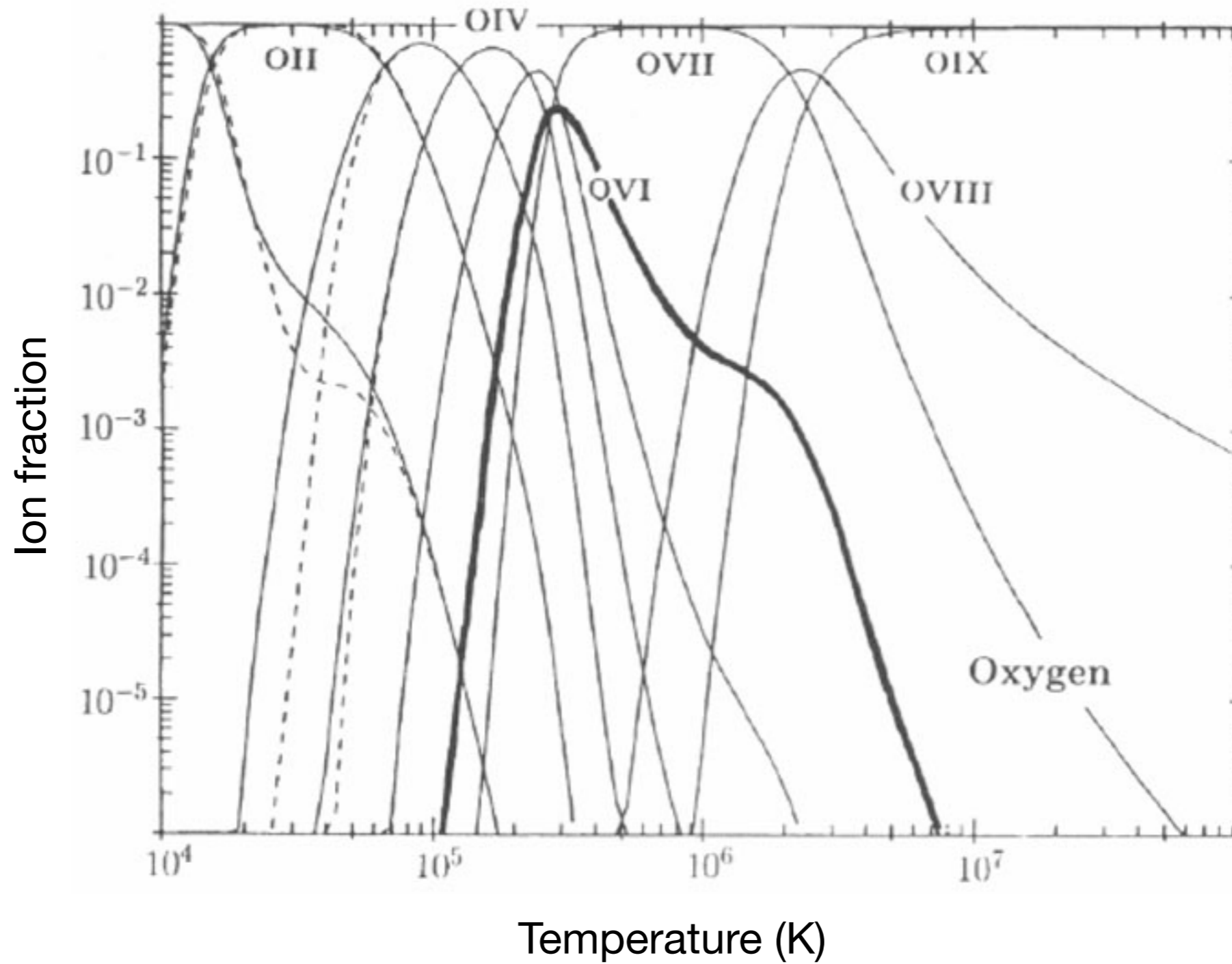
Detailed balance \Rightarrow Saha ionisation equilibrium

In general no detailed balance.

Statistical equilibrium: net upward = net downward
Rate equations to be solved to determine population

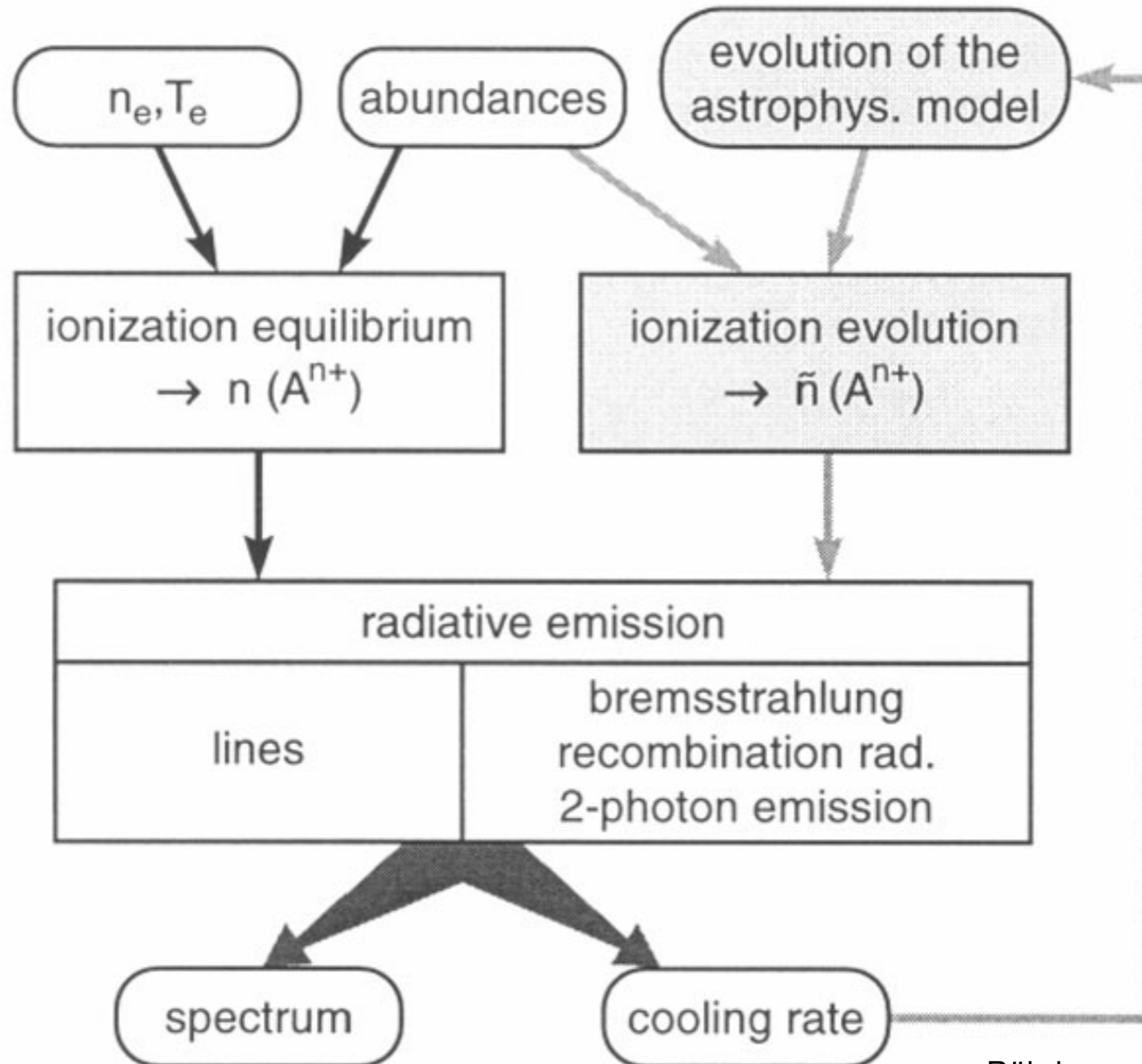
In rapidly evolving systems (e.g. young SNRs)
ionisation equilibrium may not be reached \Rightarrow *NEI*

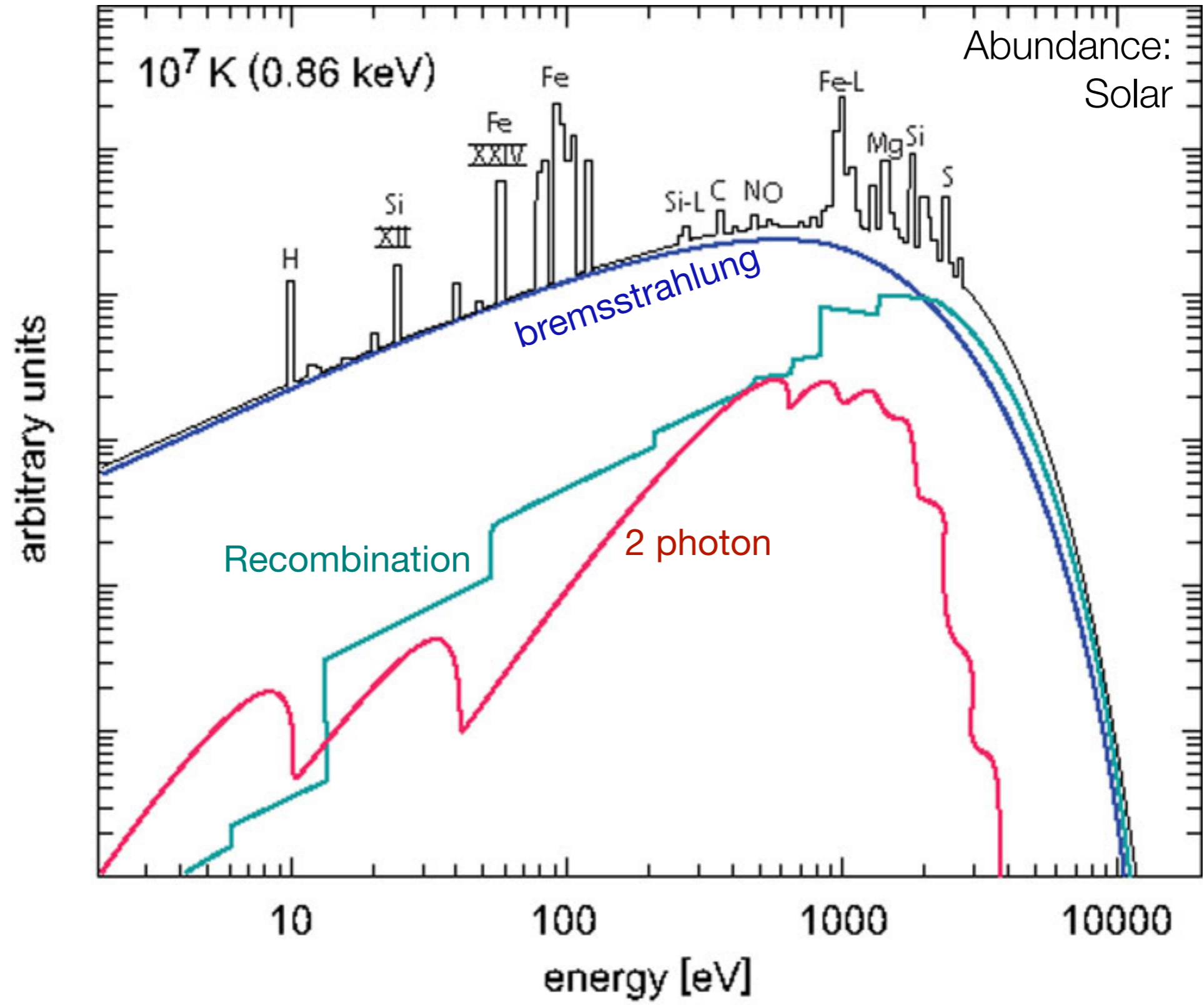
Thermal Equilibrium Ionisation

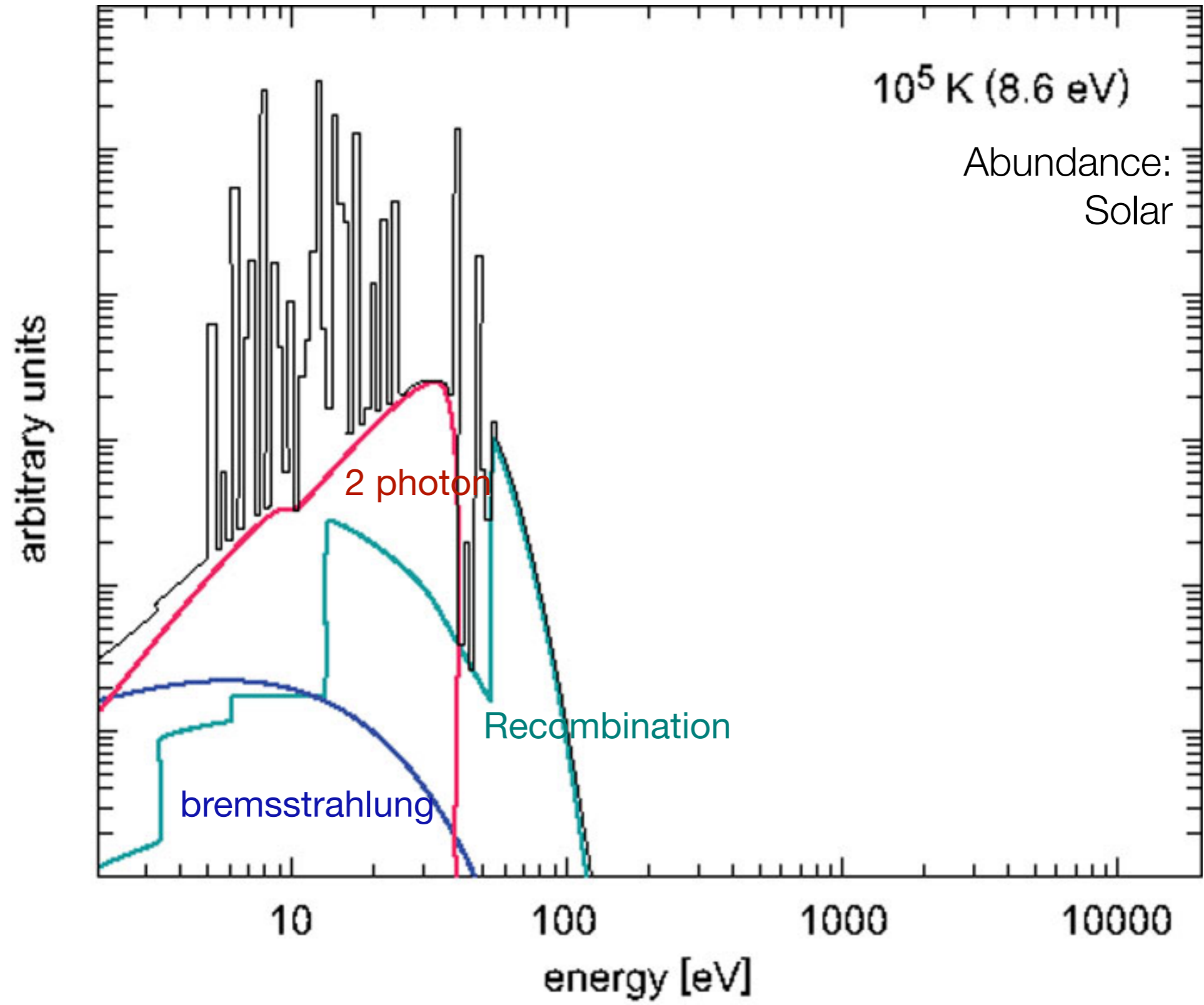


Computation of radiation from tenuous hot plasma

Transition rates from AtomDB







10^5 K (8.6 eV)

Abundance:
Solar

arbitrary units

2 photon

Recombination

bremsstrahlung

10

100

1000

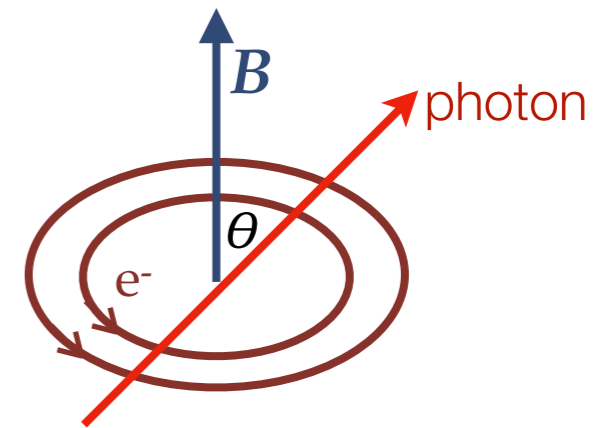
10000

energy [eV]

Electron in a magnetic field

$$\frac{p_{\perp n}}{m_e c} = \sqrt{2n(B/B_{\text{crit}})} ; n = 0, 1, 2, \dots$$

$$B_{\text{crit}} = \frac{m_e^2 c^3}{e \hbar} \sim 4.4 \times 10^{13} \text{ G}$$



$$E_n = \sqrt{m_e^2 c^4 + c^2(p_{\parallel}^2 + p_{\perp n}^2)}$$

For $p_{\parallel} = 0$ and $B \ll B_{\text{crit}}$

$$E_n - m_e c^2 = n \hbar \omega_{ce}$$

$$\hbar \omega_{ce} \sim 12 B_{12} \text{ keV}$$

Cyclotron Resonance:

$$\frac{\hbar \omega_n^{\text{res}}}{m_e c^2} = \frac{\sqrt{1 + 2n(B/B_{\text{crit}}) \sin^2 \theta} - 1}{\sin^2 \theta}$$

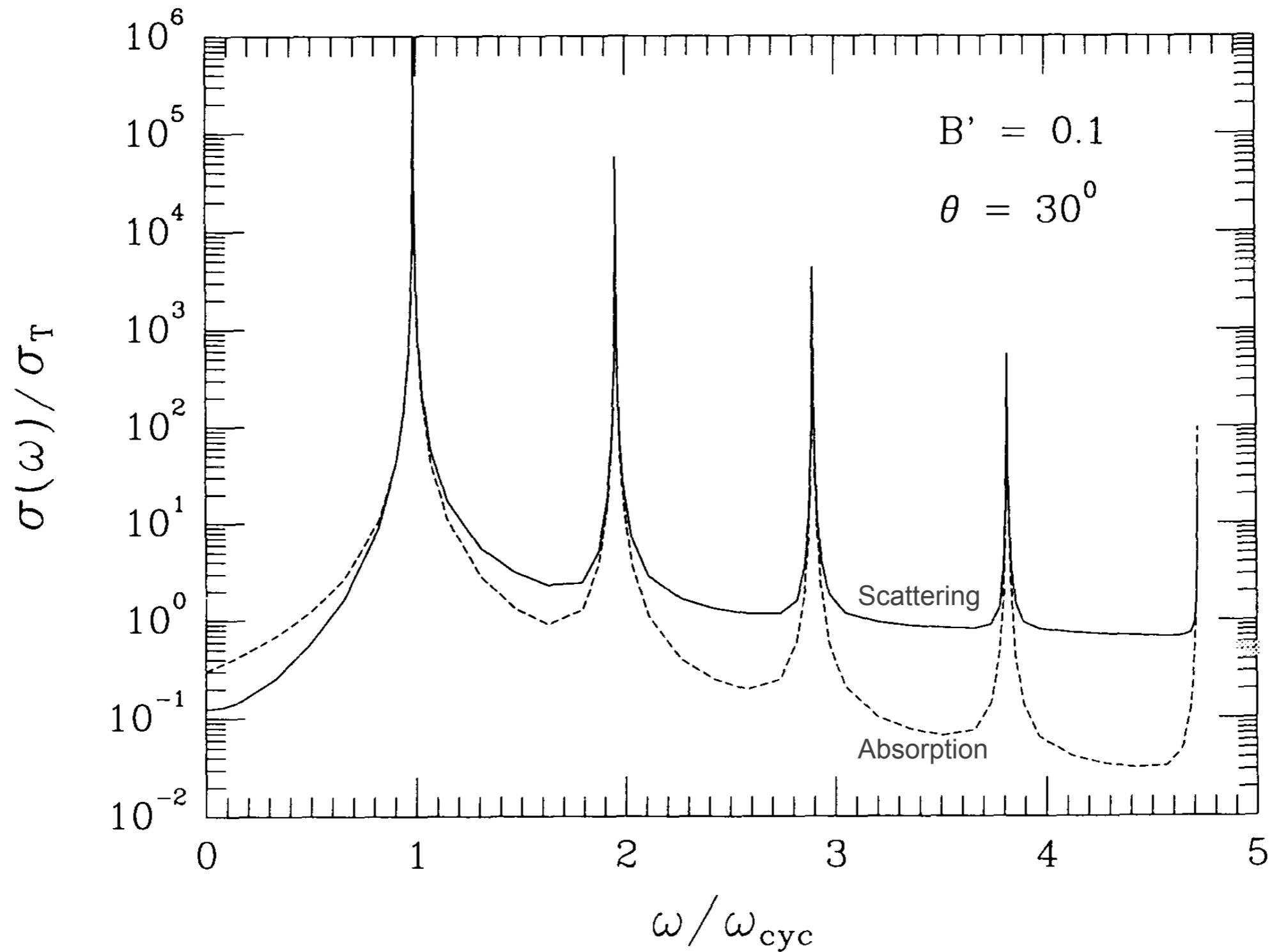
for $p_{\parallel} = 0$

Resonant processes

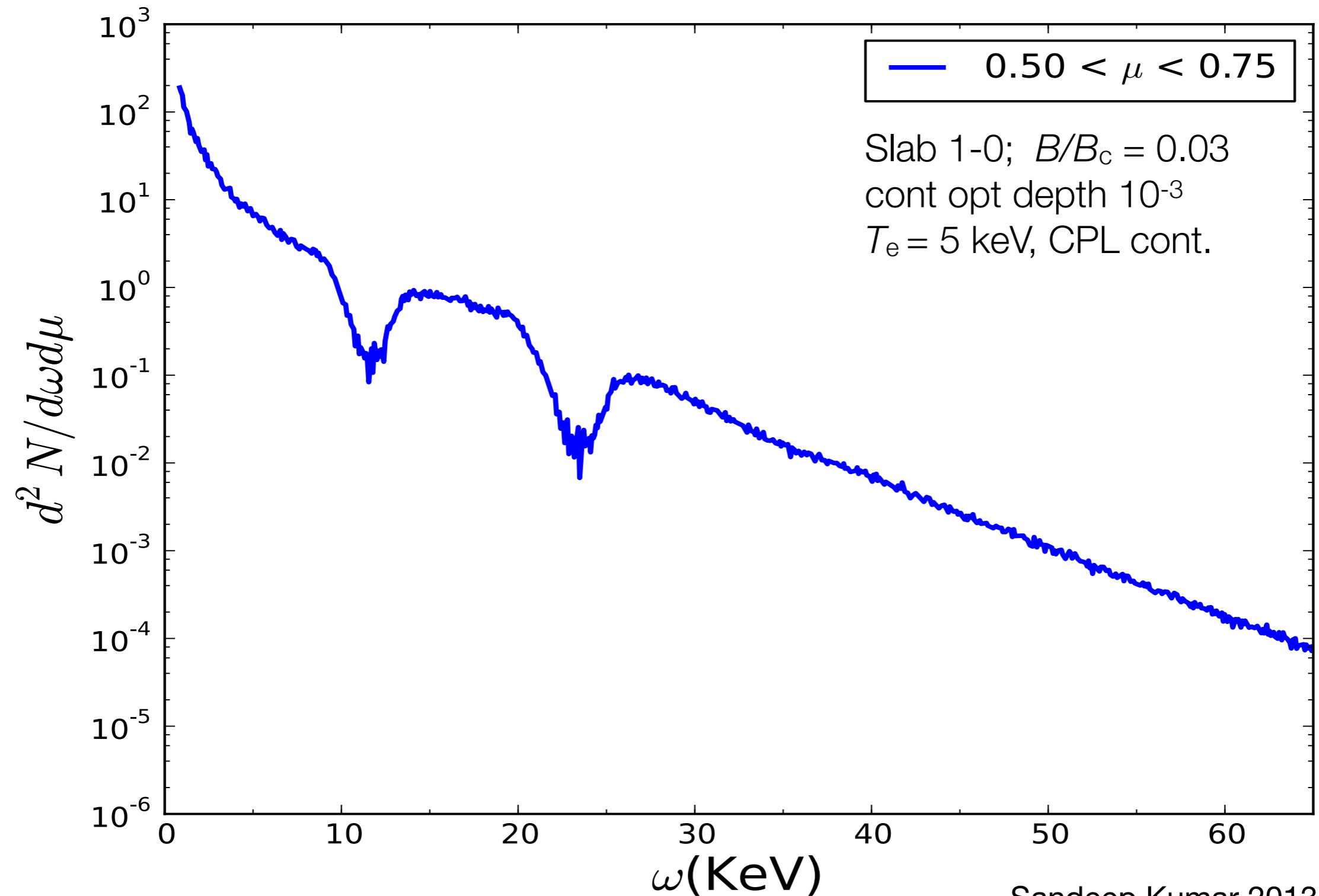
Absorption, Emission

Magneto-Compton scattering

Resonant cross sections



Cyclotron resonance spectrum computed by Monte-Carlo method



Nuclear / particle processes

Change in binding energy \rightarrow photon emission

- Radioactivity (e.g. Al^{26} 1.8 MeV)
- Decay of heavy mesons (e.g. $\Pi^0 \rightarrow 2\gamma$)
generated in nuclear scattering ($p + p \rightarrow \Pi^0$)

- Fusion

- Pair Annihilation

- Dark matter decay

