Solutions to Quiz 2

1. Give examples of $p \times p$ matrices over *integers*:
   (a) Matrices a and b that are different from each other and different from 0 such that $a \cdot b = 0$. (In other words, a and b are zero divisors.)

   Solution: We have
$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

   (b) A matrix n different from 0 such that $n^2 = 0$.

   Solution: We have
$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

   (c) A matrix p different from 0 or 1 such that $p^2 = p$.

   Solution: We have
$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

   (d) Matrices u and v different from 1 such that $u \cdot v = 1$.

   Solution: We have
$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$