Galileo II: Laws of Motion

1. Pendulum studies
2. Falling bodies/uniform acceleration
3. Projectile motion
4. Motion of the earth.

Pendulum:

Galileo first began to think about pendulum motion during a church service in Pisa. There was an oil lamp suspended from the roof of the church. He watched the priest stopped the pan of the lamp to light it and then released it with a push. The lamp began to swing from one side to the other and kept of moving due to the air current. The young Galileo, perhaps bored with the sermon, began to watch the swinging lamp. He used his pulse to keep track of how long it took the lamp to complete an oscillation. He observed that the time was constant: even when the amplitude of the oscillations decreased, it took the same time for the lamp to move from one end of its path to the other.

~~First to observe iso-chronicity of a pendulum: equal duration of oscillations for equal length of string, irrespective of the amplitude and the magnitude of the oscillating weight.~~

~~This means that the time for a pendulum to complete an oscillation will be the same even if one is made of lead and the other made of cork,~~

Even though he first got the idea as a young man watching the chandelier in the Pisa cathedral, he later developed the idea as part of his interest in falling bodies -- he saw the descent of a pendulum weight as nothing other than a fall along a circular arc.

He experimented and established that the period of swing depends only on its length, NOT on the weight of the bob, NOR the length of the arch it is swinging through. This is how he describes his own work in his later book, *Discourses Concerning Two New Sciences* (publ. 1638) which was written when he was old and under house-arrest. This book describes the work on motion that he carried out as a young professor of mathematics in Pisa. Like the earlier work, *Dialogue Concerning Two Chief World Systems*: *Ptoleamic and Copernican* (publ. 1632), the Discourses is written as a conversation between the same three characters that appear in the Dialogue, namely Salviati (who represents Galileo’s views), Sagredo (a neutral judge) and Simplicius (an Aristotelian and a simpleton).

The mass of the pendulum:

“I took two balls, one of lead and one cork, the former more than a 100 times heavier than the latter, and suspended them by two equal fine threads [of equal length]. Pulling each ball aside from the perpendicular, I let them go at the same instant… the heavy body maintains so nearly the period of the light body, that neither in a hundred swings, nor even in a thousand will the former anticipate the latter by as much as a single moment, so perfectly do they keep step. .. [the cork slows down faster.]

Isochroncity

Pendulum is not a perpetual motion machine and eventually comes to rest…the arc gets smaller and smaller with time] Each vibration, whether of 90, 50, 20, 10 or 4 degrees occupies the same time, accordingly, the speed of the moving body keeps on diminishing since in equal intervals of time, it traverses arcs which grow smaller and smaller.

The length of the thread

As to the time of vibration of bodies suspended by threads of different lengths, they bear to each other the same proportion as the square roots of length of the thread. … if one wishes to make the vibration time of one pendulum twice that of the other, he must make the suspension four times as long, nine times as long if three times the vibration time…

Pendulum clocks:

Christiaan Huygens (b. 1628), a Dutch astronomer and physicist, a contemporary of Newton, was the first to use pendulums as time-keepers in clocks. Galileo also designed a pendulum clock but by then, he was too old and ill to actually bring the project to fruition. His son later worked on Galileo’s design.

**II. Falling Bodies and non-uniform velocity (i.e. acceleration)**

the story of Galileo dropping balls of various sizes from the leaning tower of Pisa is well-known. But that was only the beginning of a long quest to understand the dynamic of free fall. (In fact, his pendulum studies were a part of his bigger interest in free fall: he saw the pendulum motion as an example of free fall).

Free-falling objects accelerate: that is, the distance that an object travels every interval of time increases as it falls to the earth. We know today that a free-falling object accelerates at the rate of 9.8 m/sec2

So, it was natural that Galileo would be interested in studying non-uniform motion, or acceleration. What is important is that Galileo subjected this complex problem to experimentation and derived laws based upon these experimental findings. (this is in contrast to Aristotelian scientists who looked for causes and purposes).

His work on motion and free fall can be understood in the following sequence:

1. Challenging Aristotle:

Aristotle held that there are two kinds of motion for inanimate matter, natural and unnatural. Unnatural (or “violent”) motion is when something is being pushed, and in this case the speed of motion is proportional to the force of the push. (This was probably deduced from watching oxcarts and boats.) Natural motion is when something is seeking its natural place in the universe, such as a stone falling, or fire rising. (We are only talking here about substances composed of earth, water, air and fire, the “natural circular motion” of the planets, composed of aither, is considered separately).

For the natural motion of heavy objects falling to earth, Aristotle asserted that the *speed of fall was proportional to the weight*, and *inversely proportional to the density of the medium* the body was falling through. He did also mention that there was some acceleration, as the body approached more closely its own element, its weight increased and it speeded up. However, these remarks in Aristotle are very brief and vague, and certainly not quantitative.

Actually, these views of Aristotle did not go unchallenged even in ancient Athens. Thirty years or so after Aristotle’s death, Strato pointed out that a stone dropped from a greater height had a greater impact on the ground, suggesting that the stone picked up more speed as it fell from the greater height.

Galileo set out his ideas about falling bodies, and about projectiles in general, in a book called “*Two New Sciences*”. The two were the science of motion, which became the foundation-stone of physics, and the science of materials and construction, an important contribution to engineering.

For example, on [**TNS page 62**](http://galileoandeinstein.physics.virginia.edu/tns61.htm), Salviati states:

*I greatly doubt that Aristotle ever tested by experiment whether it be true that two stones, one weighing ten times as much as the other, if allowed to fall, at the same instant, from a height of, say, 100 cubits, would so differ in speed that when the heavier had reached the ground, the other would not have fallen more than 10 cubits.*

Simplicio’s response to this is not to think in terms of doing the experiment himself to respond to Salviati’s challenge, but to scrutinize more closely the holy writ:

*SIMP: His language would seem to indicate that he had tried the experiment, because he says:* We see the heavier*; now the word*see*shows he had made the experiment.*

Sagredo then joins in:

*SAGR: But I, Simplicio, who have made the test, can assure you that a cannon ball weighing one or two hundred pounds, or even more, will not reach the ground by as much as a span ahead of a musket ball weighing only half a pound, provided both are dropped from a height of 200 cubits.*

This then marks the beginning of the modern era in science---the attitude that assertions about the physical world by authorities, no matter how wise or revered, stand or fall by experimental test. Legend has it that Galileo performed this particular experiment from the leaning tower of Pisa.

Galileo goes on to give a detailed analysis of falling bodies. He realizes that for extremely light objects, such as feathers, the air resistance becomes the dominant effect, whereas it makes only a tiny difference in the experiment outlined above.

*Aristotle declares that bodies of different weights, in the same medium,*

*travel (in so far as their motion depends upon gravity) with speeds which*

*are proportional to their weights; this he illustrates by use of bodies in*

*which it is possible to perceive the pure and unadulterated effect of*

*gravity****, eliminating other considerations****, for example****, figure as being of***

***small importance****,* ***influences which are greatly dependent upon the***

***medium which modifies the single effect of gravity alone.*** *Thus we*

*observe that gold, the densest of all substances, when beaten out into a*

*very thin leaf, goes floating through the air; the same thing happens*

*with stone when ground into a very fine powder. But if you wish to*

*maintain the general proposition you will have to show that the same*

*ratio of speeds is preserved in the case of all heavy bodies, and that a*

*stone of twenty pounds moves ten times as rapidly as one of two; but*

*I claim that this is false and that, if they fall from a eight of fifty or a*

*hundred cubits, they will reach the earth at the same moment.*

Galileo’s famous Thought Experiment to refute Aristotle:

Bascially this is what Galileo says: let us assume that Aristotle is correct and that the heavier of the two bodies will reach the ground first. Let us now repeat the experiment with two bodies joined together, so as to make one heavier object. This ought to fall to the ground in a shorter time than either of the two bodies – because it is heavier than either of them alone.

But it is also possible for us to reason differently: the heavier body can drag the lighter body (which is tied to it) and impart a higher velocity to it; and/or the lighter body can act as a brake on the heavier body, slowing it down a little.

Therefore, it is reasonable to expect that the fall velocity of the joined body will be intermediate of the two bodies separately. But this contradicts the Aristotelian assumption that heavier bodies will move faster.

*SALV: But, even without further experiment, it is possible to prove*

*clearly, by means of a short and conclusive argument, that a heavier*

*body does not move more rapidly than a lighter one provided both bodies*

*are of the same material and in short such as those mentioned by*

*Aristotle. But tell me, Simplicio, whether you admit that each falling*

*body acquires a definite speed fixed by nature, a velocity which cannot*

*be increased or diminished except by the use of force or resistance.*

*SIMPL: There can be no doubt but that one and the same body moving*

*in a single medium has a fixed velocity which is determined by nature*

*and which cannot be increased except by the addition of momentum*

 *or diminished except by some resistance which retards it.*

*SALV: If then we take two bodies whose natural speeds are different,*

*it is clear that on uniting the two, the more rapid one will be partly*

*retarded by the slower, and the slower will be somewhat hastened by the*

*swifter. Do you not agree with me in this opinion?*

*SIMPL: You are unquestionably right.*

*SALV: But if this is true, and if a large stone moves with a speed of,*

*say, eight while a smaller moves with a speed of four, then when they are*

*united, the system will move with a speed less than eight; but the two*

 *stones when tied together make a stone larger than that which before*

*moved with a speed of eight.* ***Hence the heavier body moves with less***

***speed than the lighter; an effect which is contrary to your supposition.***

***Thus you see how, from your assumption that the heavier body moves***

***more rapidly than the lighter one, I infer that the heavier body moves***

***more slowly.***

1. Approximation to motion in vacuum:

Aristotle had said that it was the weight of a body that caused it to fall and therefore, a heavier body will fall faster than a lighter body.

Galileo extended Archimedes idea of buoyancy to the weight problem: According to Archimedes, water support object made of material less dense than water. Galileo reasoned that air too, supports objects of small density. So, less dense objects will fall more slowly in air than denser ones. **DENSITY, not weight, was the controlling factor.** **If there were no air, all objects will fall at the same rate.**

*SALV: [. . .] We have already seen that the difference of speed between*

*bodies of different specific gravities is most marked in those media which*

*are the most resistant: thus, in a medium of quicksilver, gold not merely*

*sinks to the bottom more rapidly than lead but it is the only substance*

*that will descend at all; all other metals and stones rise to the surface*

*and float. On the other hand the variation of speed in air between balls*

*of gold, lead, copper, porphyry, and other heavy materials is so slight*

*that in a fall of 100 cubits a ball of gold would surely not outstrip one of*

*copper by as much as four fingers. Having observed this I came to the*

*conclusion that in a medium totally devoid of resistance all bodies would*

*fall with the same speed.*

*SIMPL: This is a remarkable statement, Salviati. But I shall never*

*believe that even in a vacuum, if motion in such a place were possible, a*

*lock of wool and a bit of lead can fall with the same velocity.*

Salviati rebuts Simplicio’s objection with the following argument:

since we cannot have a vacuum, let us experiment in less and less

dense media, aiming at describing the behavior in a vacuum by

extrapolation. If we find that the fall times of two different bodies get

closer and closer the less dense the medium becomes, we will then

be able to conclude that in a vacuum they would be the same. Post-

Galilean scientific research was subsequently to make widespread

use of an extrapolation procedure such as this in determining properties

that cannot be observed directly.

*SALV: A little more slowly, Simplicio. Your difficulty is not so recondite*

*nor am I so imprudent as to warrant you in believing that I have not*

*already considered this matter and found the proper solution. Hence for*

*my justification and for your enlightenment hear what I have to say.*

***Our problem is to find out what happens to bodies of different weight***

***moving in a medium devoid of resistance, so that the only difference in***

***speed is that which arises from inequality of weight****. Since no medium*

*except one entirely free from air and other bodies, be it ever so tenuous*

*and yielding, can furnish our senses with the evidence we are looking for,*

*and since such a medium is not available,* *we shall observe what*

*happens in the rarest and least resistant media as compared with what*

*happens in denser and more resistant media. Because if we find as a fact*

*that the variation of speed among bodies of different specific gravities is*

*less and less according as the medium becomes more and more yielding,*

*and if finally in a medium of extreme tenuity, though not a perfect*

*vacuum, we find that, in spite of great diversity of specific gravity, the*

*difference in speed is very small and almost inappreciable,* ***then we are***

***justified in believing it highly probable that in a vacuum all bodies***

***would fall with the same speed.***

**[BOYLE’s Air Pump; Apollo 15 hammer and a feather on the moon.** In 1971, Apollo 15th astronaut David R. Scott dropped a hammer and a feather on the surface of the moon: the two fell side by side. Scott said, “This proves Mr. Galileo was correct.” The experiment was televised and broadcast live on TV.

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1. Naturally accelerated motion:

Having established experimentally that heavy objects fall at practically the same rate, Galileo went on to consider the central question about speed of fall barely touched on by Aristotle---*how does the speed vary during the fall?*

The problem is that it’s very difficult to answer this question by just watching something fall---it’s all over too fast. **To make any kind of measurement of the speed, the motion must somehow be slowed down.**

The trick is to *slow down the motion* somehow so that speeds can be measured,*without at the same time altering the character of the motion*: even in slow-motion, the fall should have all features of a falling body.

Galileo’s idea for slowing down the motion was to have a ball roll down a ramp rather than to fall vertically. He argued that the speed gained in rolling down a ramp of given height didn’t depend on the slope, but on the vertical height. His argument was based on an experiment with a pendulum and a nail, shown on page 171 of *Two New Sciences*. The pendulum consists of a thread and a lead bullet. It is drawn aside, the string taut, to some point C.



A nail is placed at E directly below the top end of the thread, so that as the pendulum swings through its lowest point, the thread hits the nail and the pendulum is effectively shortened, so that the bullet swings up more steeply, to G with the nail at E. Nevertheless, the pendulum will be seen to swing back up to almost the same *height* it started at, that is, the points G and C are the same height above level ground. Furthermore, when it swings back, it gets up as far as point C again, if we neglect a slight loss caused by air resistance. From this we can conclude that the speed with which the ball passes through the lowest point is the same in both directions. To see this, imagine first the situation *without* the nail at E. The ball would swing backwards and forwards in a *symmetrical* way, an ordinary pendulum, and certainly in this case the speed at the lowest point is the same for both directions (again ignoring gradual slowing down from air resistance). When we do put the nail in, though, we see from the experiment that on the swing back, the ball still manages to get to the beginning point C. We conclude that it must have been going the same speed as it swung back through the lowest point as when the nail wasn’t there, because the instant it leaves the nail on the return swing it is just an ordinary pendulum, and how far it swings out from the vertical depends on how fast it’s moving at the lowest point.

Galileo argues that a similar pattern will be observed if a ball rolls down a ramp which is smoothly connected to another steeper upward ramp, that is, the ball will roll up the second ramp to a level essentially equal to the level it started at, even though the two ramps have different slopes. It will then continue to roll backwards and forwards between the two ramps, eventually coming to rest because of friction, air resistance, etc.



Thinking about this motion, it is clear that (ignoring the gradual slowing down on successive passes) it must be going the *same speed* coming off one ramp as it does coming off the other. Galileo then suggests we imagine the second ramp steeper and steeper---and we see that if it’s steep enough, we can think of the ball as just falling! He concludes that *for a ball rolling down a ramp, the speed at various heights is the same as the speed the ball would have attained (much more quickly!) by just falling vertically from its starting point to that height*. But if we make the ramp gentle enough, the motion will be slow enough to measure. (Actually, there is a difference between a rolling ball and a smoothly sliding or falling ball, but it does not affect the pattern of increase of speed, so we will not dwell on it here.)

**This means that an artificially created system can be used to study the natural phenomenon of free fall.**

1. **The Inclined plane experiments:**

This is how Galileo described his experimental set-up;

*A piece of wooden moulding or scantling, about 12 cubits long, half a cubit wide, and three finger-breadths thick, was taken; on its edge was cut a channel a little more than one finger in breadth; having made this groove very straight, smooth, and polished, and having lined it with parchment, also as smooth and polished as possible, we rolled along it a hard, smooth, and very round bronze ball. Having placed this board in a sloping position, by raising one end some one or two cubits above the other, we rolled the ball, as I was just saying, along the channel, noting, in a manner presently to be described, the time required to make the descent. We repeated this experiment more than once in order to measure the time with an accuracy such that the deviation between two observations never exceeded one-tenth of a pulse-beat. Having performed this operation and having assured ourselves of its reliability,* ***we now rolled the ball only* one-quarter *the length of the channel; and having measured the time of its descent, we found it precisely* one-half *of the former****. Next we tried other distances, compared the time for the whole length with that for the half, or with that for two-thirds, or three-fourths, or indeed for any fraction; in such experiments, repeated a full hundred times,* we always found that **the spaces traversed were to each other as the squares of the times*,*****and this was true for all inclinations of the plane,** *i.e., of the channel, along which we rolled the ball. We also observed that the times of descent, for various inclinations of the plane, bore to one another precisely that ratio which, as we shall see later, the Author had predicted and demonstrated for them.*

*For the measurement of time, we employed a large vessel of water placed in an elevated position; to the bottom of this vessel was soldered a pipe of small diameter giving a thin jet of water which we collected in a small glass during the time of each descent, whether for the whole length of the channel or for part of its length; the water thus collected was weighed, after each descent, on a very accurate balance; the differences and ratios of these weights gave us the differences and ratios of the times, and this with such accuracy that although the operation was repeated many, many times, there was no appreciable discrepancy in the results.*

**Time keeping: water clock**

 What was he able to establish:

1. The acceleration of these bodies is constant. **He demonstrated that an object released from a height starts with zero velocity and increases its speed with time** (before him it was thought that bodies when released acquire instantaneously a velocity which remained constant but was larger the heavier the object was).
2. Experimenting with inclined planes, and measuring a ball's positions after equal time intervals Galileo discovered the mathematical expression of the law of falling bodies: the distance increases as the square of the time.
3. **A falling body accelerates uniformly: it picks up equal amounts of speed in equal time intervals, so that, if it falls from rest, it is moving twice as fast after two seconds as it was moving after one second, and moving three times as fast after three seconds as it was after one second.**
4. **Galileo’s conception of Inertia:**

 Galileo reasoned that moving objects eventually stop because of a force called friction. In experiments using a pair of inclined planes facing each other, Galileo observed that a ball would roll down one plane and up the opposite plane to approximately the same height. If smoother planes were used, the ball would roll up the opposite plane even closer to the original height. Galileo reasoned that any difference between initial and final heights was due to the presence of friction. Galileo postulated that if friction could be entirely eliminated, then the ball would reach exactly the same height.

Galileo further observed that regardless of the angle at which the planes were oriented, the final height was almost always equal to the initial height. If the slope of the opposite incline were reduced, then the ball would roll a further distance in order to reach that original height.



Galileo's reasoning continued - if the opposite incline were elevated at nearly a 0-degree angle, then the ball would roll almost forever in an effort to reach the original height. And if the opposing incline was not even inclined at all (that is, if it were oriented along the horizontal), then ... an object in motion would continue in motion... .

1. Projectile motion

Galileo brought his lifetime of insight as an experimenter -- and mathematician -- to a conclusion in his greatest work, published in 1638, the *Dialogues of the Two New Sciences*. Here, in the second half of the book, he took up the question of projectile motion. This illustration

 

reflects the general opinion before Galileo which followed largely Aristotelian lines but incorporating as well a later theory of "impetus" -- which maintained that an object shot from a cannon, for example, followed a straight line until it "lost its impetus," at which point it fell abruptly to the ground. Later, simply by more careful observation, as this illustration from a work by Niccolo Tartaglia clearly shows,



it was realized that projectiles actually follow some sort of a curved path, but what sort of curve? No one knew until Galileo.

It was another essential insight that led Galileo, finally, to his most remarkable conclusion about projectile motion. First of all, he reasoned that a projectile shot from a cannon is not influenced by only one motion, but by two -- the motion that acts vertically is the force of gravity, and this pulls the projectile down by the times-squared law. But while gravity is pulling the object down, the projectile is also moving forward, horizontally at the same time. And this horizontal motion is uniform and constant according to his principle of inertia. But could he demonstrate this? In fact, by using his inclined plane again, Galileo was indeed able to demonstrate that a projectile is subject to two independent motions, and these combine to provide a precise sort of mathematical curve.

What would happen if, instead of rolling along the horizontal plane, the ball were now allowed to simply fall freely once it got to the bottom of the plane? If Galileo were correct about the horizontal and vertical motions being independent, it would still continue to move horizontally with a uniform, constant speed, but gravity would now begin to pull it down vertically at the same time, the distance increasing porportionally to the square of the time elapsed... and this is exactly what Galileo found.

1. You can see the experiment simulated in the
2. 
3. computer animation linked to the picture above.
4. You will notice how the path of the ball traces an exact curve like the one below.
5. 
6. Here is a page from one of Galileo's manuscripts in which he writes down the figures he obtained in performing this experiment himself.

 

 What he actually comes to see is that, in fact, the curve has an exact mathematical shape -- it is one the Greeks had already studied and called the parabola. The extraordinary conclusion Galileo reached in this book on the Two New Sciences is that the path any projectile follows is a parabola, and he drew exact consequences from this discovery which, as he said, could only have been achieved by the sort of exacting analysis that mathematics made possible.

Galileo’s ship:

Earth is compared to a ship; moving earth to a moving ship and it is claimed that if you were to drop a rock from a moving ship, it would fall many feet behind the mast as the ship has moved on.

G. says that those who makes such claims have not actually done the experiments that he is in fact a “better philosopher’ as he has actually made the experiment.

“when the ship is in motion, the rock does not start from rest, since the mast, the man at the top, his hand, and the rock are also moving at the same speed as the whole ship…..when the ship is moving forward, the rock also moves with the same impetus…and so it strikes the same spot where it fell when all was still.” …

Just like the ship, the moving earth carries ….

 Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though doubtless when the ship is standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. **You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still.** In jumping, you will pass on the floor the same spaces as before, nor will you make larger jumps toward the [stern](https://en.wikipedia.org/wiki/Stern) than toward the [prow](https://en.wikipedia.org/wiki/Prow) even though the ship is moving quite rapidly, despite the fact that during the time that you are in the air the floor under you will be going in a direction opposite to your jump. In throwing something to your companion, you will need no more force to get it to him whether he is in the direction of the [bow](https://en.wikipedia.org/wiki/Bow_%28ship%29) or the stern, with yourself situated opposite. The droplets will fall as before into the vessel beneath without dropping toward the stern, although while the drops are in the air the ship runs many spans. The fish in their water will swim toward the front of their bowl with no more effort than toward the back, and will go with equal ease to bait placed anywhere around the edges of the bowl. Finally the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air. And if smoke is made by burning some incense, it will be seen going up in the form of a little cloud, remaining still and moving no more toward one side than the other. **The cause of all these correspondences of effects is the fact that the ship's motion is common to all the things contained in it, and to the air also.** That is why I said you should be below decks; for if this took place above in the open air, which would not follow the course of the ship, more or less noticeable differences would be seen in some of the effects noted.