HSS 102

March 29, 2016.

Johannes Kepler and the Mind of God

Timeline

N. Copernicus 1473-1543

Tycho Brahe 1546-1601

Galileo 1564-1642

Kepler 1571-1630

*Mysterium Cosmographicum 1597*

*Astronomia Nova 1609*

*Harmonice Mundi*

## Kepler’s Significance for the Copernican revolution

1. The most obvious and well-known significance is Kepler's three laws which are even today – nearly 500 years later – are still taught in introductory science classes all around the world.

These laws can be stated as follows

1. The orbits of the planets are ellipses, with the Sun at one focus of the ellipse. 

**II. The line joining the planet to the Sun sweeps out equal areas in equal times as the planet travels around the ellipse. **

III. The ratio of the squares of the revolutionary periods for two planets is equal to the ratio of the cubes of their semimajor axes: 

These laws, and how he arrived at them, tell the story of Johannes Kepler’s life and work.

It is clear that the first two laws overthrow the Aristotelian dogma of perfect circles and uniform motion

1. Kepler is also important as a transitory figure: he was simultaneously deeply Pythagorean and Platonic in his orientation, and at the same time, deeply modern in his commitment to accuracy in observation and respect for theories fit with the data.

Kepler represents a Pythagorean orientation in physics. He believed (as did Galileo and Issac Newton) that the real is the mathematical harmony that is present in nature. The book of nature is written in the language of mathematics… knowledge of mathematical relations is the insight into the reality of the universe. He believed that God used these mathematical relations in creating the universe. To quote Peter Dear (p. 76): “ Kepler’s entire astronomical career was one directed towards gaining an understanding of God’s mind, of coming closer to God through the medium of astronomical study.”

While Pythagorean in his orientation, he was also most modern. He approached nature’s laws in the modern sense of the word as precise, empirically verifiable statements about the universal relations that govern particular phenomena.

While he was trying to figure out the workings of the “mind of God” and read the “music of the heavenly spheres,” he was ready and able to throw out his most beloved theories if they did not agree with observations.

1. He introduced physical causality into astronomy. His major work (*Astronomia Nova*) in which he describes his first and second laws is subtitled “a new astronomy based upn causation.” Before Kepler, astronomy had become an exercise in mathematical modeling of the most complicated kind including a whole bunch of fantastic ideas as epicycles: as long as the model helped in making reasonably good predictions, it was considered OK to use it, without worrying if the epicycles actually existed in reality or not.

 But Kepler was convinced that astronomy should be more than a set of mathematical devices to account for the observed phenomena. It must rest on sound physical principles as well, deriving the motions of planets from the causes producing them.

In that sense, Kepler’s astronomy was a complete reformulation in terms of aims, methods and principles. Before K, the aim of astronomers had been to produce a physics based upon circles upon circles, that could yield planetary positions that agreed with observations. K.’s goal was to find actual physical causes of the motion, that is, reasons for motions, and not merely to invent or refine geometrical schemes. Since he considered the sun to be the seat of forces, it MUST be in the PHYSICAL (and not a notional) center of the universe

## So, who was Johannes Kepler?

He was born in 1571 in the south-west Germany, close to the Black Forest. He was born in a Protestant family that had seen better days but had fallen on rough times.

His father had left to fight in the wars between the Catholics and the Protestants that were raging at that time. His mother was somewhat cranky and was accused of witchcraft in later years, and Kepler had to fight hard to save her from being burnt alive.

Kepler was born with a poor health, and had very bad eyesight, and suffered from all kinds of stomach and skin diseases.

He hardly had any formal schooling and was what we would today call a child laborer.

But fortunately for him, the local duke was a staunch Protestant who believed very strongly in raising well-educated Christians who could stand their grounds against the Catholics. So, Kepler was admitted in his seminary where he studied to become a priest and later graduated from University of Tubingen; but luck intervened, and he was offered a post of a teacher of mathematics and astronomy in Gratz, in Austria.

He was introduced to Copernicus’s sun-centered model by Michael Maestlin, his teacher at the University of Tubingen. Maestlin used to give private lessons to his more promising students. In these private sessions, he used to teach Copernicus, while in public lectures at the university; he used to teach the old Ptolemaic model.

Kepler embraced the Copernican system and remained a firm Copernican all his life. He embraced it even though there was no evidence for helio-centrism at this point. He just found the model beautiful and more agreeable to his Christian beliefs. He thought that the Sun, being the noblest of all heavenly bodies HAD to be in the center, just like God is the center of all creation.

### Kepler’s work in Gratz:  *Mysterium Cosmographicum* (*the Cosmic Mystery*)

This book was published in 1597, when JK was barely 26 years old teacher in Gratz, Austria.

A year after his arrival in Gratz – on July 9, 1595, for JK recorded the date—he was drawing a figure on the blackboard for his class, when an idea struck him with such force that he felt that he was holding the key to the secrets of the universe. This idea was to determine the course of his life and work.

The idea was this: that the universe is built around certain symmetrical figures – triangles, squares, pentagon etc. – which form the skeleton of the universe. This idea was actually completely WRONG, but it was to inspire Kepler to the work that led him to his three laws.

Kepler then began to ask a rather odd (by modern standards) question: “*why* were there just six planets instead of twenty or a hundred” and *why* the distances and velocities of the planets were what they were. He was also fascinated by the way Copernicus had arranged the orbits of planets. Unlike Ptolemy’s model, the Copernican model had a logic to it that made it harmonically whole. It **required** the planets to be located at precise distances from the Sun in this order:

The orbit of Mercury had to be one third the orbit of Earth

The orbit of Venus had to be two-third the orbit of Earth

The orbit of Mars had to be 1.5 times the orbit of Earth

The orbit of Jupiter had to be five times the orbit of Earth

The orbit of Saturn had to be ten times the orbit of Earth

All these ideas were in Kepler’s mind when he drew a simple picture like this on the blackboard:



 This picture was basically a simple diagram of a triangle fitted between two circles, with the three points of the triangle touching the outer circle and the three sides of the triangle enclosing the inner circle. **Because Kepler had been thinking hard of Copernican model, when he saw this diagram, he began to see some details that may not be visible to others: He found that the ratio of the sizes of the larger and smaller circle were very close the ratios of orbits of Saturn and Jupiter, as required by the Copernican model.**

This simple drawing led Kepler to find the solution to his problem, i.e., why there were only six planets (at that time, including the earth, which following the Copernican system, he counted among the six planets, i.e., mercury, venus, earth, Mars, Jupiter and Saturn.)

He reasoned that there were six planets, because there were only FIVE REGULAR SOLIDS and THEREFORE, God created six planets so that they could fit the five regular solids between them (just as he had fitted a triangle between two circles in the classroom).

What are regular solids? They are three dimensional figures, each side of which is an identical two-dimensional figure.

There are only five regular solids: (recall our Plato lecture)

1. Cube: six faces, each being a square
2. Tetrahedron: four faces, each being an equilateral triangle
3. Octahedron: eight faces, each being an equilateral triangle
4. Do-decahedron: 12 faces, each being a pentagon
5. Icosahedron: 20 faces, each being an equilateral triangle.

So this is what Kepler did:

He began to nest these perfect solids in spheres representing the orbits of the planets;

* He took one sphere, approximating the size of the orbit of the most outermost planet, Saturn (#1)
* He inserted a cube within it
* He inserted a sphere (#2, the orbit of Jupiter) within the cube
* Inserted a tetrahedron within the sphere #2
* Inserted a sphere (#3, orbit of Mars)
* Inserted a do-decahedron within the sphere #3
* Inserted a sphere (#4, orbit of earth)
* Inserted a Icosahedron within sphere 4
* Inserted a sphere (#5 , orbit of Venus)
* Inserted a octahedron within the sphere 5
* Inserted a sphere (#6, orbit of mercury)

The resulting models looked like this

# http://upload.wikimedia.org/wikipedia/commons/thumb/1/19/Kepler-solar-system-1.png/220px-Kepler-solar-system-1.png File:Kepler-solar-system-2.png

Note that the size of the sphere we begin with will determine the size of the cube we place in it, which in turn will decide the size of the sphere that will fit the cube and so on. But although the actual distance between each sphere depends upon the choice of the first sphere, the RELATIVE distance between the sphere will depend upon the size of the regular solids. So no matter what size sphere we begin with, the relative distance between the spheres will remain the same.

Kepler found to his great delight that the relative distance between the planetary orbits that he found after nesting all these regular solids and spheres was pretty close to the distances that had been calculated from the Copernican model.

The young math teacher of Gratz was ecstatic: he felt he had figured out the mind of God! He argued that it could not be just a chance that there were only five perfect solids and only five intervals between the six planets. God must have intended it so. This arrangement also explained why the distances between the planets were what they were: Planets HAD to be placed in such a manner that the five solids could be exactly fitted into the intervals, as an invisible skeleton of the universe.

But the difference between Kepler (a transitory figure between the medieval and the modern era) is that unlike the natural philosophers of the previous generations, he did not rest by merely discovering the “mind of God” – he went about testing it against the known sizes of the orbits and the distances between the planets. He discovered that while there was fairly good agreement between his model and the known data for the orbits of Mars, Earth and Venus, the data did not agree with his model for Jupiter and Mercury.

Kepler was quite upset, but he did not discard his model: he blamed Copernicus’s bad data for the lack of fit.

Since he had no access to astronomical instruments of that era – quadrants, cross bars etc. –he could not make the observations himself. So began his quest for better observational data that will prove the accuracy of his model that finally led him to Tycho Brahe, the world’s most renowned naked-eye astronomer of this period. (more on him below)

## Reception of Kepler’s *Mysterium*

Kepler’s old teacher, Michael Maestlin, encouraged Kepler to publish the book and helped him in the process of getting the book into print. Kepler sent out copies to all the well known astronomers – including Galileo (who ignored it, pretty much) and to Tycho Brahe, who responded warmly as he saw the creative mind behind it.

Kepler also got the duke of Wuerttenberg (his home town) interested in a project to build a three dimensional model of his model of the universe. There were great plans to build a huge model in gold, with precious stones for planets etc. Kepler spent six months creating a three dimensional model with paper for the goldsmiths to follow. But nothing really came out of this project.

In the mean time, there were political problems in Gartz where Kepler, being a Protestant came under political pressures. He had to quit his job and move to a safer place.

He wrote to Tycho for help and ended up working as his assistant (and later occupied Tycho’s office as the imperial mathematician in the service of the king of Prague.)

At this point, it is important to bring Tycho Brahe into the story.

## Who was Tycho de Brahe

If Kepler was a pauper, Tycho was born with a silver spoon in his mouth. He was born to a very rich Dutch family with all kinds of titles and estates.

As was common with the upper crust, he was being trained to become an emabassdor or a statesman. He was sent to the University of Copenhagen for studies.

But while he was a freshman, he happened to witness a partial eclipse of the moon – and even that had been predicted in advance. He was totally taken in by the fact that it was possible to predict celestial events by understanding the motion of stars and the planets.

Since then, astronomy became his passion and he began collecting all kinds of astronomical instruments and books. His family tried to dissuade him, but he had chosen his vocation.

## Tycho’s observatory at Hveen:

Tycho soon established a reputation for his highly rigorous and accurate observations of the sky. He was given an entire island – the island of Hveen – by the king Frederick II of Denmark to set up an observatory. There Tycho set up a workshop, hired experienced craftsmen to build him the finest astronomical instruments – astrolabes, quadrants and the like.

He later had to leave the island because of the problems he created by mistreating the peasants.

He was later invited by the king of Prague, Rudolph II, to become his imperial mathematician/astrologer and to create new star tables, to be called the Rudolphine tables in his honor. It is Prague that Kepler will meet Tycho.

## Tycho’s contributions to astronomy

1. De Nova Stella (the New Star) was Tycho’s first book published in 1573. It described Tycho’s observations of a new star.

On the evening of Nov 11, 1572, Tyco was walking home from a friends’ laboratory when he noticed a new star in the constellation of Cassiopeia (the familiar W shaped cluster of stars). This star was brighter than others and remained in the same position for 18 months.

Tyco turned his fancy instruments on it and made rigorous observations.

He showed beyond any doubt that it was a fixed star way beyond the moon in the celestial regions which was supposed to be changeless according to the Aristotelian dogma. He ruled out the possibility that it could be a comet in the sub-lunary region.

This was a great challenge to Aristotle and Ptolemy.

1. He also made similar observations on the comet of 1577. He followed the path of the comet and showed that it was passing exactly where one of the crystalline spheres of Ptolemaic astronomy was supposed to be! This was another assault on Aristotle’s picture of the cosmos. (Kepler also saw this comet: he was only a child of 5 at this time and his mother took him up a hill to show him the bright new star. )
2. Tychos model:

For all his brilliance as an astronomer, Tycho was unable to accept the idea of a moving earth : he could not overcome the traditional objections to earth’s motion. So he was unable to accept Copernicus.

He used his data to construct his own model which combined the most appealing features of Ptolemy and Copernicus.

In his model, the earth was in the center of the universe. The moon and the sun revolve around the earth, but the planets revolve around the Sun. Thus, although the earth is stationary and at the center of the universe, the center of movement of the planets is the Sun (as in the Copernican system.) (see diagram on the next page)



The Tychonic System

Kepler and Tycho:

Kepler met Tycho in Prague on Feb 4, 1600.

Kepler was then 29 years old and Tycho 53 years of age.

Kepler, as mentioned above, was fleeing religious persecution, penniless with this wife and children with him.

Tycho hired him as an assistant and set him to work on the planet Mars which was/is one of the most difficult planets to study due to the shape of its orbit.

Tycho and Kepler met with cross-purposes in mind:

Tycho was hopeful that Kepler will use his creative mind to synthesize the vast amount of data he had to support his, i.e., Tychonic model

But Kepler was hoping to use the fabulous wealth of Tycho’s data to prove *his* own model right.

Within a year and a half of their meeting, Tycho died due to some trouble with his kidneys. His dying words were “let me not seem to have lived in vain.” He asked Kepler, who was with him at the time of his death, to prove the truth of the Tychonic model and make him famous. But that is not how it turned out…

Within two days of Tycho’s death, Kepler was offered the job of the Royal Mathematician in the service of the king of Prague. Kepler, naturally ,accepted and stayed in that post from 1601 to 1612. He did his most productive work there.

## Kepler’s first two laws

Kepler published his second book – which he considered his major work and contained his first two laws of planetary motion – in 1609. The book is called *Astronomia Nova* (or *A New Astronomy*). The complete title read:

A New Astronomy

Based upon causation

Derived from the Investigations of the star Mars

Founded on observations of the Noble Tycho Brahe

It is in this book that Kepler lays down his first two laws (see the beginning of the lecture).

The story of how he arrived at these laws is truly heroic and shows a genius at work.

He started with the conventional beliefs about heavenly bodies moving in perfect circles at uniform speed. With the first two laws, he threw out both of these beliefs that had dominated astronomy for nearly 2000 years: he concluded that planets move NOT in circles but in ellipses (the First law) and that they move NOT at uniform speeds but in a manner that a line drawn from the planet to the sun always sweeps an equal area.

When he started his observations of Mars, his objective was to describe the orbit of Mars by

* Calculating the radius of the orbit
* Determine the axis joining the prehelion (the point nearest to the sun) and aphelion (the point farthest from the sun).

How he proceeded:

From Tycho’s vast collection of observational data of Mars, he selected four positions when the planet was in opposition to the sun in 1587, 1591, 1593 and 1595. From these four positions, he had to determine the radius of the orbit and the direction of the axis.

This required patient and painstaking calculations:

He has left behind some 900 folio pages of calculations, which took him some 5 years to perform.

After heroic efforts, he came up with a solution.

But even after such heroic efforts, he decided to throw away his results and start afresh. The reason was that when he used his figures for the radius and the axis to calculate the positions of mars which he had chosen from Tycho’s data, there was a discrepancy of **eight minutes of an arc.**

 He was willing to throw away five years of hard work for 8’ arc: this was unprecedented, as astronomers of that era were used to much bigger discrepancies. But Kepler insisted that since Tycho had left behind a treasure of very accurate data, it was incumbent upon him to be fastidious in his calculations as well.

 So he went back to the Square One , as it were.

He realized that it was simply impossible to construct a CIRCULAR orbit that would satisfy all existing observations. It dawned upon him that maybe he will have to find some other kind of curve to explain the observations. He had to face the data without any preconceived notions about the shape of the orbit.

He also figured that in order to understand Mars’ motion, he will first have to understand the earth’s motion better, because after all, we are viewing mars from the earth.

So he deviced a mathematical method of transferring the observer’s positon from earth to mars, and to compute the motion of earth as an observer from Mars will see it.

He discovered that the earth, like the other planets, does not revolve at uniform speed, but moves faster or slower depending upon the distance from the sun. He also found out that at two extreme positions of the orbit (the aphelion and prehelion) the earth’s velocity was inversely proportional to the distance from the sun.

Because Kepler believed in real physical forces that moved the planets, he imagined a force emanating from the sun (like the light that emanates from the sun). This force, he believed, acted like a broom sweeping the planets around in their orbit. As the sun revolved around its axis, the force which it emitted also rotated with it, and like the spokes of a wheel, swept the planets along.

From this kind of causal reasoning, kepler tried to arrive at a formula to predict the position of the earth at any given time, given the fact that it did not move at a uniform speed and that its speed varied inversely form the distance from the sun.

This led him to the second law of areas: he concluded that the area swept by the line connecting the planet to the sun is a measure of the time required by the planet to get from the point A to B., hence the line will sweep equal areas in equal time. This is Kepler’s second law.

The second law determined the variations of the planet’s speed along its orbit, but did not determine the shape of the orbit. He tried circle once again, but gave up as the data did not fit a circle. He finds the oval shape best accounts for the observed positions. After many attempts at trying to figure out the exact shape of this egg-shaped orbit, Kepler undersood that he was dealing with a perfect ellipses. This led him to his first law of elliptical motion.

## Harmonica Mundi

This was Kepler’s third major book. It was published in 1619, a period of a lot of struggle and strife in his life which saw the death of his daughter and his wife, and witchcraft charges against his mother.

In this work, he tries to fit perfect polygons within the spheres and concludes that there are certain harmonies between the spheres that can be detected.

This book contains his Third Law. But apart form that, this work on the music of the spheres is merely a historical curiosity, with not much scientific significance.

Other works:

His *Epitome of Copernican Astronomy* appeared in three volumes produced in 161, 1620 and 1621.

In 1627, Kepler completed the Rudolphine star tables, named after his employer, the King Rudolph of Prague. The tables made it possible to calculate the position of planets to an accuracy 30 times that of the tables Copernicus had produced. These tables remained in use for many generations after Kepler. Indeed, a later astronomer, a French man by the name of Pierre Gassendi observed the transit of Mercury (when Mercury passes in front of the sun) which had been predicted by Kepler’s tables.

Kepler also dabbled in a bit of Biblical history. Going by the eclipse recorded in King Herod’s time, he calculated that Jesus was actually born five years before the common era.

Kepler passed away on Nov. 15, 1630, a few weeks shy of his 59th birthday.