Solutions to Quiz 3

(5 marks) 1. A Player rolls two dice repeatedly until the numbers shown on them are different.

1. Write down the probability of the event A_n that there are n throws and sum on the final (n-th) throw is 7.
2. What is the probability of $B_n = \cup_{k=0}^{n} A_k$?

Calculate the probability that the final throw gives a sum of 7.

Solution: Let E_i be the event that on the i-th throw the numbers are the same. Let D_i be the event that on the i-throw the sum is 7. We note that $P(E_i) = 1/6$ and $P(D_i) = 1/6$ as well. Then $A_0 =$ (no throws and so no 7!)

$$A_n = E_1 \cap \cdots \cap E_{n-1} \cap (E_n^c \cap D_n)$$

Note that D_n and E_n are mutually exclusive, so we also have

$$A_n = E_1 \cap \cdots \cap E_{n-1} \cap D_n$$

Since the different throws are independent we have $P(A_0) = 0$ and $P(A_n) = (1/6)^n$ for $n \geq 1$.

By the argument like the one above we see that A_n's are all mutually exclusive, hence $P(B_n) = \sum_{k=1}^{n} (1/6)^n$.

The probability that the final throw gives a sum of 7 is the same as the probability of $\cup_n B_n$. By the limiting law of probability, this is $\sup_n P(B_n)$ which is $\sum_{n=1}^{\infty} (1/6)^n$. We calculate this

$$\sum_{n=1}^{\infty} (1/6)^n = (1/6) \sum_{m=0}^{\infty} (1/6)^m = (1/6) \frac{1}{1-(1/6)} = (1/6) \cdot (6/5) = 1/5$$