Quiz 8: Products

Question

Consider the category $\mathcal{T}$ whose objects are topological spaces morphisms are continuous maps. Find the fibre-product (if it exists) of $(1, 3) \to \mathbb{R}$\$ and $(0, 2) \to \mathbb{R}$ where these are the natural inclusions of these intervals as subspaces in the space of real numbers with the usual topology.

Answer

Given a space X with continuous maps $f : X \to (1, 3)$ and $g : X \to (0, 2)$ such that composed with the above inclusion gives the *same* continuos map $h : X \to \mathbb{R}$.

For every point $x \in X$, this means $f(x) = g(x) = h(x)$. Thus, $1 < f(x) < 3$ and $0 < g(x) < 2$ which means that $1 < h(x) < 2$. Thus, we get that the h maps X to $(1, 2)$ such that f and g are the composition of this map (denote it as $h_1 : X \to (1, 2)$) with the inclusions $(1, 2) \to (1, 3)$ and $(1, 2) \to (0, 3)$ respectively.

It follows easily that the fibre-product is $(1, 2)$ with the above inclusions as the two maps.

Note The above generalises to *any* two subspaces of a topological space: The fibre-product of the inclusions is the intersection of these two subspaces.