

Biplab Das

Unable to understand about homogeneous coordinate in projective geometry.

homogenize \rightarrow

$$x_1^2 + x_2^3 + x_3 = 5$$
$$x_0 x_1^2 + x_2^3 + x_0^2 x_3 - 5 x_0^3 = 0$$

$x_1 \rightarrow x_1/x_0$ $x_2 \rightarrow x_2/x_0$ $x_3 \rightarrow x_3/x_0$

$x_2^3 \Rightarrow$
 $x_2 = 0$

Asymptotic points

a line at infinity.

$\rightarrow x_0 = 0 \rightarrow$ points at ∞ !

Biplab Das

Unable to find any preciseness here. Will you please elaborate this today.

General case

$$\varphi: E[x, y] \rightarrow F$$

$$E \rightarrow F$$

$$x \rightarrow a \text{ transsec}$$

Working inductively over finitely many elements a_1, \dots, a_d of F , we see that we can re-order them so that $E(a_1, \dots, a_d)$ is of the form $E(b_1, \dots, b_t)[c_1, \dots, c_u]$ where:

- ▶ b_i is transcendental over $E(b_1, \dots, b_{i-1})$, and
- ▶ c_j is algebraic over $E(b_1, \dots, b_t)[c_1, \dots, c_{j-1}]$.

Here $b_i = a_{\sigma(i)}$ and $c_j = a_{\sigma(t+j)}$ for some permutation σ of $1, \dots, d$.

$$\frac{a_1, \dots, a_d}{E(a_i)}$$

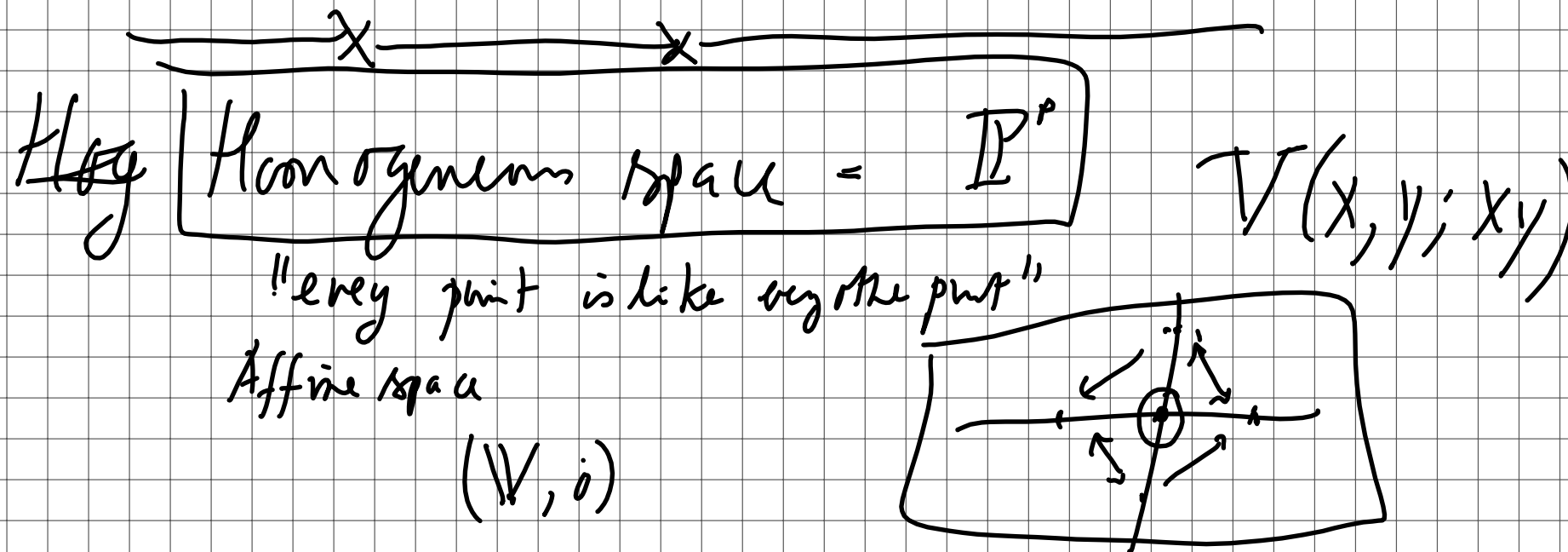
Sayan Chattopadhyay

This is primarily motivated from one of the discussions that happened in class.

One can think of the difference between affine space and vector space in another way. Consider a vector space V , this comes with an additive group of the vector space (which I also denote by V). So one defines an affine space A associated to this vector space as the set of "points" on which the group V acts by translation (this is free and transitive), and given two points in the affine space, say p, q , there is a unique vector w of V such that $p - q = w$

So in a certain sense, this construction of the affine space shows how an affine space forgets the origin and only remembers the difference between two distinct points. A naive motivation would be to think of the affine space as physical points and the vector space associated as direction vectors (maybe velocities?) between points. A concrete concept here is that of G -torsor, where G is a group. An affine space is then a V -torsor

I was wondering if there could be such a similar interpretation of the projective space as some kind of a torsor?



Prakash Joshi

the main Problem with me in following this week lecture's is that i can not understand what exactly an affine scheme is . by its definition as in class ;Z- affine scheme is simply a collection of polynomial f_1, f_2, \dots, f_q in variable x_1, x_2, \dots, x_p .

means it is simply a subset of $Z[x_1, \dots, x_p]$. so why we considering it seprately and not as a subset. so what is intrsting point for defing this?

$X = A(x_1, \dots, x_p; f_1, \dots, f_q)$ - (conceptually we think of solutions of $f_1 = \dots = f_q = 0$ of the form (a_1, \dots, a_p) lying in some ring $R = \mathbb{R}$

"Shape" of a subset of \mathbb{R}^p $R = \mathbb{F}_7$
Geometry

Geometry is understood by looking at algebraic morphisms. $X \rightarrow Y$ for various Y

Groups (G, e, i, μ) - discrete.

Topological group \supset Algebraic group.

\mathbb{Q}
 \mathbb{C}
 \mathbb{Z}

$X =$ "Totality of R -points for every R "

$R \subset S$

$X(R) \subset X(S)$

$\mathbb{F}_p \not\subset \mathbb{Z}$

$X(\mathbb{F}_p) \not\subset X(\mathbb{Z})$

① Is \mathbb{Z} - \mathbb{Z} affine scheme is just a way of ~~notating~~ notation. ✓

since its definition says nothing its collection of polynomial variables x_1, x_2, \dots, x_p in \mathbb{Z} .

simply saying that is its subset of polynomial ring $\mathbb{Z}[x_1, x_2, \dots, x_p]$.

② Can you in lecture ④

we try to understand \mathbb{Z} -point.

$X(\mathbb{R})$

Can you explain it again when $\mathbb{R} =$ finite field F_q

over the field \mathbb{F}_p

where $q = p^r$

Kirti Taneja

Can you elaborate about solutions of algebraic equations in finite rings? When we look at p-tuples of commuting matrices that satisfy the system of equations, can we gain some additional information (by looking at trace or other properties of matrices)?

$\mathbb{F}_{27} \leftrightarrow \mathbb{F}_3$ Algebraic properties of rings.

We can't draw a "picture"

\mathbb{R} or \mathbb{C} can draw a "picture"

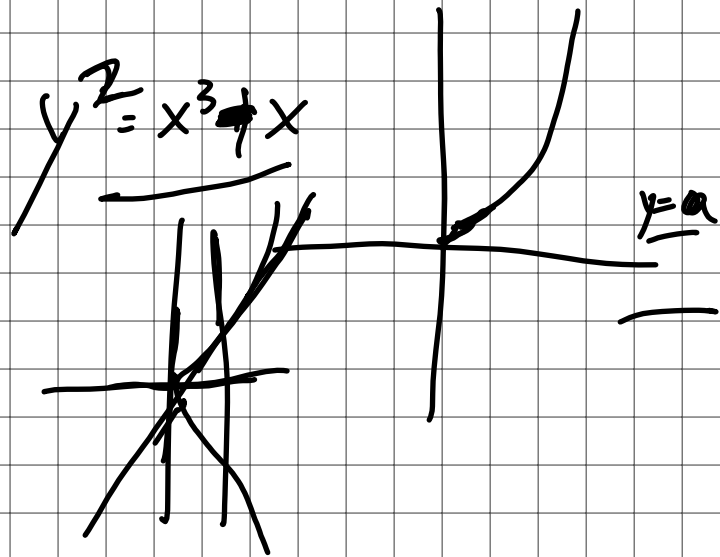
Line $\mathbb{C}A(x, y, z, w; xy - zw = 0)$ $\lambda(a, b, c, d)$

$A(x, y, z, w; xy - zw = 1)$ (also contains lines.

$$\mathbb{R}.p = \mathbb{L} \subset \mathbb{R}^4$$

$x=a \quad z=b \quad ay - bw = 1$

Ruled Surfaces = "curved" surfaces made up of lines.



(counting points of
intersection)

Different points
behave differently