**Inequalities**

Justify all your answers.

1. In each pair of numbers below, which is larger? Explain why.

(1 mark) (a) 10/81, 11/90

(1 mark) (b) 100/811, 111/900

2. In each pair of numbers below, which is larger? Explain why.

(1 mark) (a) $(10)^5$, $10000 \cdot (11)^2$

(1 mark) (b) $(100)^5$, $10000 \cdot (101)^2$

(1 (bonus)) (c) $n^5$, $10000 \cdot (n+1)^2$ for large $n$.

3. Give two positive rational numbers $p/q$ and $r/s$ (this means that $p$, $q$, $r$ and $s$ are natural, or counting, numbers). Suppose that $p/q < r/s$, which of the following numbers lies in between? (*Hint*: If you can't do it right away, try putting values for the variables to help you.)

(1 mark) (a) $\frac{(p/q)+(r/s)}{2}$

(1 mark) (b) $\sqrt{(pr)/(qs)}$

(1 mark) (c) $(p+r)/(q+s)$

(1 (bonus)) (d) Order the above three numbers.

4. Given that $p$ and $q$ are counting numbers so that $p^2 > 3q^2$ and put $r/s = (2p+3q)/(p+2q)$. Show that:

(1 mark) (a) $r^2 > 3s^2$

(1 mark) (b) $r/s < p/q$

(1 (bonus)) (c) Use *this* idea to find a rational number $a/b$ so that $100(a^2 - 3b^2) < b^2$.

(*Hint*: If you can't do it right away, try putting values for the variables to help you.)