Solutions to Quiz 6

1. Find the Fourier series for the following function:

$$f(x) = \begin{cases} x & 0 \leq x \leq \pi \\ 0 & -\pi \leq x < 0 \end{cases}$$

(Show the steps of your integration. Do not write the answer from memory!)

Solution: We calculate

$$a_0 = \frac{1}{\pi} \int_0^\pi x dx = \frac{\pi}{2}$$

(1 Mark for this.)

Next, we have

$$a_n = \frac{1}{\pi} \int_0^\pi x \cos(nx) dx$$

We integrate by parts (1 Mark for this)

$$a_n = \frac{1}{\pi} \int_0^\pi x d\left(\frac{\sin(nx)}{n}\right) = \frac{1}{\pi} \int_0^\pi d\left(x\frac{\sin(nx)}{n} + \frac{\cos(nx)}{n^2}\right)$$

So one evaluates (1 Mark for this)

$$a_n = \frac{1}{\pi}\left(x\frac{\sin(nx)}{n} + \frac{\cos(nx)}{n^2}\right)\Bigg|_{x=0}^{\pi} = \frac{1}{\pi}\frac{(-1)^n - 1}{n^2}$$

Similarly, we have

$$b_n = \frac{1}{\pi} \int_0^\pi x \sin(nx) dx$$

We integrate by parts (1 Mark for this)

$$b_n = \frac{-1}{\pi} \int_0^\pi x d\left(\frac{\cos(nx)}{n}\right) = \frac{-1}{\pi} \int_0^\pi d\left(x\frac{\cos(nx)}{n} - \frac{\sin(nx)}{n^2}\right)$$

So one evaluates (1 Mark for this)

$$b_n = \frac{-1}{\pi}\left(x\frac{\cos(nx)}{n} - \frac{\sin(nx)}{n^2}\right)\Bigg|_{x=0}^{\pi} = \frac{(-1)^{n+1}}{n}$$

Putting it all together the Fourier series is

$$\frac{\pi}{4} - \frac{2}{\pi}\sum_{k=0}^{\infty}\frac{\cos(2k+1)x}{(2k+1)^2} + \sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin(nx)}{n}$$