Solutions to Quiz 4

(5 marks) 1. For each of the following differential equations decide whether it is exact. If it *is* exact, then find the solution.

(a) $\cos(y)dx + e^x dy = 0$.

Solution: We have $(M, N) = (\cos y, e^{-x})$. We check

$$\frac{\partial M}{\partial y} = -\sin y \neq -e^{-x} = \frac{\partial N}{\partial x}$$

Thus, this differential is not exact. (1 Mark)

(b) $(x + y)dx + (x - y)dy = 0$.

Solution: We have $(M, N) = (x + y, x - y)$. We check

$$\frac{\partial M}{\partial y} = 1 \neq= \frac{\partial N}{\partial x}$$

Thus, this differential *is* exact. (1 Mark)
We now calculate (1 Mark):

$$\int M dx = x^2/2 + xy$$

Next we calculate (1 Mark):

$$N - \frac{\partial}{\partial y}\int M dx = (x - y) - x = -y$$

Finally, we get (1 Mark):

$$\Phi(x, y) = \int M dx \int + \left(N - \frac{\partial}{\partial y}\int M dx\right) = x^2/2 + xy - y^2/2$$