Find your match

(The following is based on a section in the book "Asymptopia" by Joel Spencer and Laura Florescu.)

We have 52 gamblers who are given the following game.

- First of all, each gambler is given one card from a Blue pack of 52 distinct cards.
- There is a room with a table. The game master arranges 52 cards from a Red pack of cards into 4 rows of 13 cards each. (The cards have been shuffled, and put face down, and the game master has noted the exact position of each card.)
- Each gambler must go into the room and look at the face of 26 Red cards. If the gambler finds the matching card Red Card to the Blue card she/he already has, then she/he has won his/her round.
- After the gambler leaves the room, the Red cards are put back *exactly* as before she/he came in. Moreover, the gambler cannot meet the other gamblers after leaving the room and takes the Blue card away.
- The gamblers are allowed to come up with a common strategy at the beginning.

The question is: "What is the best strategy for the gamblers?" "Do the chances of winning change if we only have 40 gamblers and leave out all the"picture" cards?" "What are the optimal probabilities?"

Some hints:

- If the gamblers do not employ any strategy and each one looks at 26 cards at random, then each has 1/2 chance of winning his round.
- You should use group theory!