Solutions to Quiz 1

1. Let $\mathcal{C}_0$, $\mathcal{C}$, ℓ_1 and ℓ_∞ denote the usual spaces of sequences. Give examples of each of the following:

(1 mark) (a) An element of $\mathcal{C}_0$ which is not in ℓ_1.

> **Solution:** The sequence $(1, 1/2, 1/3, \ldots)$ converges to 0 but is not absolutely summable.

(1 mark) (b) An element of $\mathcal{C}$ which is not in $\mathcal{C}_0$.

> **Solution:** The sequence $(1, 1, 1, \ldots)$ converges to 1.

(1 mark) (c) An element of ℓ_∞ which is not in $\mathcal{C}$.

> **Solution:** The sequence $(1, -1, 1, -1, \ldots)$ is bounded but does not converge.

(2 marks) (d) Elements $a^{(n)}$ of $\mathcal{C}_0$ for $n = 1, 2, \ldots$ so that $a_k^{(n)}$ is Cauchy for each fixed k but the sequence of elements is not Cauchy in $\mathcal{C}_0$.

> **Solution:** We take $a^{(n)} = (0, \ldots, 0, \overset{n}{\check{1}}, 0, \ldots)$ which has 0 in all places except 1 in the n-th place. The sequences $a_k^{(n)}$ for a fixed k consist of 0's for $n > k$ and so are Cauchy. However, $||a^{n)} - a^{(m)}|| = 2$ whenever $n \neq m$. So the sequence is not Cauchy in $\mathcal{C}_0$.