Probability and Statistics
MTH202 First Mid-Sem Exam 9 February 2018

Solutions

1. Three fair coins C_1, C_2, C_3 are flipped simultanously. If the faces seen are all the same (i.e. all three heads or all three tails), then the three coins are flipped simultaneously again, otherwise we stop. Let W denote the random variable denoting the number of times that the coin C_1 is flipped. The random variable S denotes the number of Heads that we see when we stop. Calculate the probabilities indicated and answer the question.

(1 mark) (a) $P(W = n)$

(a) $(2/8)^{n-1}(6/8) = 3/(4^n)$

(1 mark) (b) For what value of n is $P(W = n)$ the highest?

(b) $n = 1$ **for which** $P(W = 1) = 3/4$

(1 mark) (c) $P(S = 2)$

(c) $3/6 = 1/2$

(1 mark) (d) $P(W = 4 \wedge S = 1)$

(d) $(2/8)^3(3/8) = 3/(2^9)$

Actually, the variables W and S are independent!

(1 mark) (e) $P(W \geq 3)$.

(e) $(2/8)^2 = 1/2^4$

$P(W \geq 3)$ is the probability that the first two flips result in all heads or all tails.

2. Provide the probabilities for each of the following (in each case assume that "randomly" means that each possible choice is equally likely):

(1 mark) (a) A randomly chosen number between 0 and 1699 (both included) is divisible by 17.

(a) $|\{0, 17, 34, \ldots, 1683\}|/1700 = 100/1700 = 1/17$

(1 mark) (b) Given that 45% of students at IISER Mohali are female, what is the probability that a randomly chosen student is male?

(b) **55/100=11/20**

(1 mark) (c) Given $P(A) = 1/2$ and $P(B) = 1/3$ and that A and B are mutually exclusive, what can you say about $P(A \wedge B)$?

(c) $P(A \wedge B) = 0$ **since the events are mutually exclusive**

(1 mark) (d) Given $P(A) = 1/2$ and $P(B) = 2/3$ is it possible that A and B are mutually exclusive?

(d) **No;** $P(A \wedge B) = P(A) + P(B) - P(A \vee B) \geq 1/2 + 2/3 - 1 = 1/6$

(1 mark) (e) Given $P(A) = 1/4$ and $P(B) = 1/3$ and that A and B are independent, what can you say about $P(A^c \wedge B^c)$?

(e) $P(A^c \wedge B^c) = (3/4) \cdot (2/3) = 1/2$

3. The number of cars of different makes and colors on IISER Campus is given in the following table:

Make	Dark	White	Silver	Other
Maruti	5	10	5	10
Hyundai	1	2	2	2
Honda	1	1	1	1
Toyota	0	3	1	0
Other	2	2	2	1

In each of the questions below, assume that each car is equally likely to be picked.

(1 mark) (a) What is the probability that it is a Maruti Car?

(a) $30/(30 + 7 + 4 + 4 + 7) = 15/26$

(1 mark) (b) Given that the car is White in colour, what is the probability that it is a Maruti Car?

(b) $10/(10 + 2 + 1 + 3 + 2) = 5/9$

(1 mark) (c) Given that the car is a Maruti car, what is the probability that the car is White?

(c) $10/(5 + 10 + 5 + 10) = 1/3$

(1 mark) (d) Given that the car is *not* a Maruti car, what is the probability that it is White?

(d) $(2 + 1 + 3 + 2)/(7 + 4 + 4 + 7) = 4/11$

(1 mark) (e) Given that the car is *not* a Maruti car and *not* Dark, what is the probability that it is white?

(e) $(2 + 1 + 3 + 2)/(6 + 3 + 4 + 5) = 4/9$