Coins, Dice, etc.

1. A four-sided dice (with faces showing different numbers) is rolled three times. What is the probability that the numbers obtained are different?

2. Player A flips a coin 12 times with the hope of getting at least 2 heads. Player B rolls dice 6 times with the hope of getting at least 1 six. Which player has a better probability of success?

3. Suppose we have a collection of $100n$ samples, which are uniformly distribution among 100 values so that each value is taken *exactly* n times. Calculate the mean, median, mode and variance.

4. A standard (6-sided) dice is rolled two times. Let X_i denote the value obtained on the i-th roll. Is this a random variable? What about $S = X_1 - X_2$, the difference of the values rolled? What are the distributions of each of these random variables?

5. A player has a fair coin, and a standard 4-sided dice and a standard 6-sided dice. The player flips the coin, if it shows head, then the 4-sided dice is rolled and its value noted; else if the coin shows tail then the 6-sided dice is rolled and it value noted. Is the result X, a random variable? What are the values of $P(X = i)$?

6. A die is rolled repeated until we get a 6. The number of rolls is recorded. Let X denote the random variable that denotes the number of rolls.

   1. Calculate the value of X for which the probability is the highest.
   2. What is the smallest s so that $P(X \leq s) \geq 1/2$?

7. A fair dice is rolled until we see a 6. What is the probability that we never see a 6?

8. A fair dice is rolled until we see a number different from 6. Let F be the random variable that is the number that we see. What is the probability distribution of F?

9. A fair dice is rolled and a fair coin is flipped. If we see a tail, then we take the value of the dice; if we see a head then we add 6 to the value of the dice *unless* we see a 6; if we see a head and a 6 then we repeat the experiment. What is the probability distribution of the random variable X which we obtain as the value?

10. In a "random" chemistry experiment (we do not have precise control over the outcome) we can observe whether the solution is acidic (event A) and whether the solution is coloured (event B). Assume that $P(B) > 0$. We now carry out the experiment repeatedly until B is observed. Express the probability that A is observed at the same time as B in terms of the probabilities of events A and B and their unions and intersections.

11. We know that 60% of the dogs in the campus are black. We repeatedly observe dogs until we find one that is black; in that case we record its age before releasing it. After 20 such recordings we find that 5 of these black dogs were puppies. Assuming that frequency

is a good estimate of probability, what is a reasonable estimate of the percentage of black puppies in the campus dog population? Can we conclude the 25% of the dogs are puppies?