Power Series and Derivatives

1. Write the Taylor Series with remainder term of the following functions.

(2 marks) (a) $\sin(x)/\cos(x)$ till 4 terms.

(2 marks) (b) $\exp(x)/(1+\exp(x))$ till 4 terms.

(2 marks) (c) $\sin(|x|^3)$ (as many terms as possible).

(2 marks) (d) Given continuously differentiable functions f and g such that $f(0) = 1$, $g(0) = 0$, $df/dx = g$ and $dg/dx = f$. Write the Taylor series of f and g and try to recognise them assuming that they are determined by the Taylor series.

(2 marks) (e) $\sin(x) + \cos(x)\exp(-1/x^2)$ (where, by convention we treat $\exp(-1/x^2)$ as 0 at 0).