

Limits

1. Find the limit superior and the limit inferior of the following sequences. (You may use limit calculations from the notes.)

(1 mark) (a) $\left(1 + \frac{1}{n}\right)_{n \geq 1}$

(1 mark) (b) $\left((-1)^{n+1} + \frac{(-1)^n}{n}\right)_{n \geq 1}$

(1 mark) (c) $\left(\left(1 + \frac{1}{n}\right)^n - \left(1 + \frac{1}{n}\right)\right)_{n \geq 1}$

(1 mark) (d) $\frac{n-2}{n^2+2n+1}$

(1 mark) (e) $\frac{n^2 - (-1)^n 2n - 1}{n^2 + 2n + 1}$

- (1 (bonus)) (f) For each n , let k_n be such that 2^{k_n} is the *smallest* power of 2 which is greater than n ; in other words $2^{k_n-1} \leq n < 2^{k_n}$. Now take the sequence $(n/2^{k_n})_{n \geq 1}$.

(1 (bonus)) (g) $(\sin(n))_{n \geq 1}$.

2. Show that the following sequences have a limit.

- (1 mark) (a) The sequence $(x_n)_{n \geq 1}$ where

$$x_n = 1 + \frac{2}{n} + \frac{3}{n^2}$$

- (1 mark) (b) The sequence $(x_n)_{n \geq 1}$ where

$$x_n = \frac{1 + 2\frac{1}{n} + 3\frac{1}{n^2}}{1 - 2\frac{1}{n} + 3\frac{1}{n^2} - 4\frac{1}{n^3}}$$

- (1 mark) (c) The sequence $(x_n)_{n \geq 1}$ where

$$x_n = 1 + 2\left(5 + \frac{1}{n}\right) + 3\left(5 + \frac{1}{n}\right)^2$$

- (1 mark) (d) The sequence $(x_n)_{n \geq 1}$ where

$$x_n = \frac{1}{1 + 2\left(5 + \frac{1}{n}\right) + 3\left(5 + \frac{1}{n}\right)^2}$$

(1 mark) (e) $\left(\left(1 + \frac{1}{n+1}\right)^n\right)_{n \geq 1}$