Sequeces and Series

1. Which of the following series is convergent and which diverges to infinity?

(1 mark)   (a) $\sum_{n=1}^{\infty} \frac{1}{n+20}$

(1 mark)   (b) $\sum_{n=1}^{\infty} \frac{n+1}{n^2}$

(1 mark)   (c) $\sum_{n=0}^{\infty} \frac{x^n}{n+1}$ for $0 < x < 1$.

(1 mark)   (d) $\sum_{n=1}^{\infty} x^{n-n^2}$ for $0 < x < 1$

(1 mark)   (e) $\sum_{n=1}^{\infty} x^{n^2-n}$ for $0 < x < 1$

(1 mark)   (f) $\sum_{n=0}^{\infty} \frac{x^n}{n^4}$ for $x > 1$.

(1 (bonus))   (g) $\sum_{n=1}^{\infty} n \cdot x^n$ for $0 < x < 1$.

2. Which of the following sequences is *eventually increasing* and is *bounded above*?

(1 mark)   (a) $1 - \frac{n+1}{2n^2-n}$

(1 mark)   (b) $\left(1 + \frac{n}{n^2+1}\right)^n$

(1 mark)   (c) The sequence with $x_1 = 1$ and

$$x_{n+1} = \frac{2x_n + 1}{x_n + 1}$$

(1 mark)   (d) The sequence with $x_1 = 1$ and

$$x_{n+1} = \frac{2x_n + 3}{x_n + 2}$$