Sequences

Justify all your answers.

1. Compare the following sequences to decide which one is *eventually* larger.

(1 mark) (a) The sequence with general term 10^{15}

$$10^{15}, 10^{15}, 10^{15}, \ldots$$

versus the sequence with general term $\frac{n}{10^{15}}$

$$\frac{1}{10^{15}}, \frac{2}{10^{15}}, \frac{3}{10^{15}}, \ldots$$

(1 mark) (b) The sequence with general term $n \cdot 10^{10}$

$$1 \cdot 10^{10}, 2 \cdot 10^{10}, 3 \cdot 10^{10}, \ldots$$

versus the sequence with general term $n^2/10^{15}$

$$\frac{1^2}{10^{15}}, \frac{2^2}{10^{15}}, \frac{3^2}{10^{15}}, \ldots$$

(1 mark) (c) The sequence with general term $(n + 1000) \cdot 10^{10}$

$$(1001) \cdot 10^{10}, (1002) \cdot 10^{10}, (1003) \cdot 10^{10}, \ldots$$

versus the sequence with general term $n^2/10^{15}$

$$\frac{1^2}{10^{15}}, \frac{2^2}{10^{15}}, \frac{3^2}{10^{15}}, \ldots$$

(1 mark) (d) The sequence with general term $n \cdot 10^{10}$

$$1 \cdot 10^{10}, 2 \cdot 10^{10}, 3 \cdot 10^{10}, \ldots$$

versus the sequence with general term $(n^2 - n)/10^{15}$

$$0 = \frac{0}{10^{15}}, \frac{2}{10^{15}}, \frac{6}{10^{15}}, \ldots$$

(1 (bonus)) (e) The sequence with general term 2^n

$$2^1, 2^2, 2^3, \ldots$$

versus the Fibonacci sequence with general term $F(n) = F(n-1)+F(n-2)$ starting with $F(1) = 5$ and $F(2) = 8$.

$$5, 8, 13, \ldots$$

2. Give the properties of each sequence out of:

$$\text{eventually increasing}, \text{eventually decreasing}, \text{neither}, \text{bounded}, \text{unbounded}$$

(1 mark) (a) The sequence with general term $n/(n+1)$.

$$1/2, 2/3, 3/4, \ldots$$

(1 mark) (b) The sequence with general term $5(n+1)^2/(n^3+2n)$

$$20/3, 45/12, 80/33, \ldots$$

(1 mark) (c) The sequence with general term $n^{100}/2^n$

$$1/2, 2^{98}, 3^{100}/8, \ldots$$

(1 mark) (d) The sequence with general term $2^{2n-1}/(10^n)$

$$2/10, 8/100, 32/1000, \ldots$$

(1 (bonus)) (e) The sequence with general term $n \cdot \sin(1/n)$

$$\sin(1), 2\sin(1/2), 3\sin(1/3), \ldots$$

3. Which of the following sequences has an *upper* bound and which does not.

(1 mark) (a) The sequence with general term $n^2 - 100 \cdot n$.

(1 mark) (b) The sequence with general term $1000 \cdot n^2 - 2^n$.

(1 (bonus)) (c) The sequence with general term $1 + 1/2 + \cdots + 1/n!$ where $n! = 1 \cdot 2 \cdots n$ is the factorial of n.