

The Number System

The process of learning often starts with the realisation that we need to look more closely at something we think we understand. In the process, we find that some aspects are not as clear as we originally thought. We then think a little more and, in resolving our doubts, we understand it better!

With this philosophy in mind will now look at numbers more closely since mathematics often begins with numbers. Do we really understand numbers in the usual way we use them?

Counting Numbers

The first numbers we learn about are counting numbers which can be used to count objects: one student, two students etc. We want to count because we generally believe that bigger numbers indicate some sort of progress. However, this is not always the case: a thousand students in a class is often not better than a hundred students in a class, at least for the grader of examinations!

Counting can be done with “tick” or “tally” marks. However, it becomes difficult to “name” larger collections. (It is said that in early humans counting was limited to one, two, three and many!) We need introduce some sort of bunching system. The standard one is to draw a diagonal across four tally marks as the fifth tally mark. This way we count bunches of “fives” and then a “left over” number (less than five). For example, a count of eleven is represented as $\text{||||} \text{||||} |$.

This does not solve the problem since we may have a lot of bunches! We will then need to tally up bunches and in doing that we would need to create bunches of bunches. For even larger numbers we would need bunches of bunches of bunches and so on! So, the *place* system was invented (probably in India) as a way to represent large numbers. In the decimal place system we create bunches of tens rather than fives. The tally of “left overs” is in the last place of decimal, then comes the “left over” tens in the next place, and so on. If there are no “left overs” in a particular place, we need a “place-holder” and so the symbol 0 is invented for that.

This completes our high-level pseudo-historical overview of the decimal system of counting! However, it may be a good idea for the reader to think about why the usual “methods” (algorithms) that we use to add and multiply decimal numbers are correct in terms of this interpretation. We will re-visit this later when we look at polynomials.

Measuring