Partial Differential Equations

1. Solve the classical 1-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$$

for $x \in [0, \pi]$ under the conditions

$$u(0,t) = u(\pi,t) = 0 \text{ and } u(x,0) = f(x) \text{ and } \frac{\partial u}{\partial t}(x,0) = 0$$

where f is the function given below. (In this question and subsequent questions you can make use of the Fourier series calculated in other assignments.)

$$f(x) = \begin{cases} x & 0 \leq x \leq \pi/2 \\ \pi - x & \pi/2 \leq x \leq \pi \end{cases}$$

2. Solve the classical 1-dimensional heat equation

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$

for $x \in [0, \pi]$ and $t \geq 0$ under the conditions

$$u(0,t) = u(\pi,t) = 0 \text{ and } u(x,0) = |\sin(2x)|$$

3. Solve the classical 2-dimensional Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

for $x^2 + y^2 \leq 1$ under the conditions

$$u(x,y) = x^2 \text{ for } x^2 + y^2 = 1$$