## Solutions to Quiz 5

1. Find one solution of the equation

$$x^2y'' + x(3+x)y' + y = 0$$

using the Frobenius method.

**Solution:** We substitute $y = \sum_k y_k x^k$ with $y_k = 0$ for $k$ sufficiently negative.

Now we equate powers of $k$ to get the equations

$$(k(k-1) + 3k + 1)y_k + (k-1)y_{k-1} = 0$$

(1 Mark for this equation.)

It follows that

$$y_k = -\frac{(k-1)}{k^2+2k+1} y_{k-1}$$

As long as the denominator is non-zero. Now

$$k^2 + 2k + 1 = (k+1)^2$$

So as long as $k \neq -1$, $y_k$ is a multiple of $y_{k-1}$. (1 Mark for this).

Since $y_k = 0$ for sufficiently negative $k$, we see that the only possible non-zero $y_k$ are for $k = n-1$ for $n$ a non-negative integer. (1 Mark for this.)

We put $y_{n-1} = a_n$ and obtain

$$a_n = -\frac{(n-2)}{n^2} a_{n-1} \text{ for } n \geq 1$$

(1 Mark for this.)

So we get

$$y = a_0 x^{-1} (1 + x)$$

(1 Mark for this.)