Fourier Series

1. Find the Fourier series for the following functions:

   (a)
   $$f(x) = \begin{cases} 1 & 0 \le x \le \pi \\ 0 & -\pi \le x < 0 \end{cases}$$

   (b)
   $$f(x) = \begin{cases} \sin(x) & 0 \le x \le \pi \\ 0 & -\pi \le x < 0 \end{cases}$$

   (c)
   $$f(x) = \begin{cases} x^2 & 0 \le x \le \pi \\ 0 & -\pi \le x < 0 \end{cases}$$

2. Use the answers in the previous exercise to calculate the Fourier series for the following functions:

   (a)
   $$f(x) = \begin{cases} 1 & 0 \le x \le \pi \\ -1 & -\pi \le x < 0 \end{cases}$$

   (b)
   $$f(x) = \begin{cases} \sin(x) & 0 \le x \le \pi \\ -\sin(x) & -\pi \le x < 0 \end{cases}$$

   (c) $f(x) = x^2$ for $x \in [-\pi, \pi]$.

   (d) $f(x) = x|x|$ for $x \in [-\pi, \pi]$.

3. Given $a < b$ in $[-\pi, \pi]$, Find the Fourier series for the functions given below:

   (a)
   $$f(x) = \begin{cases} 1 & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

   (b)
   $$f(x) = \begin{cases} x & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

4. Use the previous exercise with scaling and linearity to find the Fourier Series of any piecewise linear function.