Solutions to Quiz 3

1. A fair die is flipped 180 times. Let S be the random variable that counts the number of 6's.

(1 mark) (a) What is the expectation $E(S)$?

> **Solution:** We have probability of success $p = 1/6$ and number of Bernoulli trials is $N = 180$. Thus $E(S) = Np = 30$.

(1 mark) (b) What is the variance $\sigma^2(S)$?

> **Solution:** We have probability of success $p = 1/6$ and number of Bernoulli trials is $N = 120$. Thus $\sigma^2(S) = Np(1-p) = 25$.

(2 marks) (c) Use Chebychev's inequality to bound the probability that S lies in the range $[11, 29]$.

> **Solution:** Note that the original question was supposed to be $[21, 39]$ and the problem was mis-typed!
> Given $a = 2$, we get $a\sigma(S) = 10$. By Chebychev's inequality
>
> $$P(|S - 30| \geq 10) \leq 1/a^2 = 1/4$$
>
> Thus $P(|S - 20| < 10) \geq 1 - 1/4 = 3/4$.
> Chebychev's inequality only gives lower bounds for the probability of region that are symmetric around the mean and contain it.
> Hence, for the given range it does *not* give any information! So full marks will be given for anyone who correctly states Chebychev's inequality in this context!