Estimates and Limit Distributions

1. Calculate the following expressions and the error in comparison with the target expression.
   (a) $\sum_{k=0}^{5} \frac{1}{3^k}$ for the target $1/(1-\frac{1}{3})$.
   (b) $\sum_{k=0}^{3} \frac{1}{k!}$ for the target $\exp(1)$.
   (c) $\sum_{k=0}^{4} \frac{1}{4^{k+1}(k+1)}$ for the target $-\log(1-\frac{1}{4})$.

2. Suppose we want to calculate the value of $\log(2)$ using the formula
$$\log(2) = \sum_{k=0}^{\infty} \frac{1}{2^{k+1}(k+1)}$$
   How many terms will we use to calculate it correctly to 2 places of decimal? (Note that the error should be at most 1/200; justify this statement!) Use a calculator to calculate this value and compare it with the built-in value.

3. Using the above value of $\log(2)$ and the following formula
$$\log(n+1) - \log(n) = \sum_{k=0}^{\infty} \frac{1}{(n+1)^{k+1}(k+1)}$$
   to calculate $\log(3)$, $\log(5)$ accurate upto two places of decimal. (note that $\log(4) = 2\log(2)$!) Compare it with the built-in value on your calculator.

4. A die is tossed 500 times and S is the random variable that counts the number of 6's that occur.
   (a) Write a formula for $p_k = P(S = k)$, for $m = E(S)$ and $s = \sigma(S)$.
   (b) Write a formula for the probability of that the number of 6's that we throw is betweem $m - 3s$ and $m + 3s$.
   (c) Assuming that we can use the normal distribution with mean m and standard deviation s to approximate this probability write an expression for it.

5. A fair coin is flipped 1000 times and the number of heads is recorded. Give an integral formula for the probability that the number of heads lies in the range $[480, 520]$. Also write the estimate for this probability using the Chebychev formula.

   Suppose that we want to estimate the probability that the number of heads lies in the range $[490, 510]$. What is the problem with the estimate using Chebychev's inequality?

6. Out of a collection of 500 samples of DNA, 10 are known to have a mutation. An experimentalist is checking these samples *one-by-one* for the presence of the mutation. Give an estimate for the smallest number of samples needed to be checked so that the experimentalist can have a probability of at least 4/5 (or 80%) of seeing the mutation.

7. Suppose that the probability of success in a trial is 1/50. Write a formula for the probability of 2 successes in 20 trials. Also apply the discrete Poisson distribution to get an estimate of the probability. Note how the two values differ. What will happen to the difference if the probability of success is in a single trial is 1/100)?

8. Suppose that a shooting star (meteorite) passes overhead once per hour on average. What is the probability that you watch for one hour and see no shooting star? How often should you look at the sky so that your probability is within 1% of this probability?

9. Suppose that the frequency of occurrence of an certain scintillation is f times per second. What is the probability that you will have to wait for t seconds before you see a scintillation? What is the expected waiting time for a scintillation?

10. Two hundred students eat in the mess in a period of two hours on average. Assume that the students enter the mess at random times and that the mess runs continuously for all time! What is the probability that no student will enter the mess over a period of ten minutes? What is the amount of time that one should wait to be 90% certain that a student will enter the mess during that time?

11. An buzzer goes with a frequency of f times per hour. We observe the buzzer for a period of one hour. What is the probability that the buzzer will go off k times? What is the expected number of times the buzzer will go off?

12. About 10 planes pass low overhead in IISER Mohali campus very day from 9:00 Hours to 19:00 hours on average. We have a sound sensitive experiment that takes 30 minutes to carry out. Assume that the experiment is completely ruined if 3 (or more) planes pass overhead during the experiment. What is the probability that an experiment is completely ruined?

13. A chef adds 200 cashew pieces to 2 kilogrammes of rawa idly *maavu*. Each idly weighs 100 grammes before cooking. Assuming that kaju pieces weigh nothing (they are so light!), what is the probability of finding at most 1 kaju piece in an idly? What is the expected number of kaju pieces in an idly?