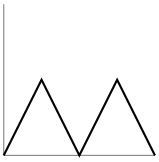
Solutions to Quiz 2

(2 marks) 1. (Lab-based) The shape of the histograms of X and Y are given by the charts below.

(X) 

(Y) 

Answer the following questions.

(1 mark) (a) In which case is the median above the centre?

> **Solution:** Clearly Y is symmetric and so the median is in the centre. In the case of X, the first half clearly has smaller area so the median is above the centre.

(1 mark) (b) In which case is the variance larger?

> **Solution:** In the case of Y, the mean is again at the centre (due to symmetry) and the counts fall off towards the side. On the other hand, the mean for X is somewhere in the middle and the counts *rise* away from it to the right. This makes the variance larger.

2. We are given a discrete random variable W that takes only non-negative integer values $k = 0, 1, 2, \ldots$. We are given that $P(X = k) = 1/3^k$ *only for* $k > 0$. What are the following probabilities?

(1 mark) (a) $P(X > 2)$

> **Solution:** Since the events $P(X = k)$ are mutually exclusive, the probability that $P(X > 2)$ is the sum of the probabilities $P(X = k)$ for $k > 2$. This is (using the geometric series)
>
> $$\sum_{k=3}^{\infty}(1/3^k) = (1/3)^3\frac{1}{1-(1/3)} = (1/3^3)\cdot(3/2) = 1/18$$

(1 mark) (b) $P(X = 0)$

> **Solution:** By the same reasoning as above we see that $P(X > 0)$ is
>
> $$\sum_{k=1}^{\infty}(1/3^k) = (1/3)\frac{1}{1-(1/3)} = (1/3)\cdot(3/2) = 1/2$$

Since $P(X = 0)$ and $P(X > 0)$ are exclusive *and* exhaustive, it follows the

$$P(X = 0) = 1 - P(X > 0) = 1 - (1/2) = 1/2$$