Solutions to Quiz 4

1. Sita and Gita go to the village fair where there is a game where each has 2/5-th chance of winning one round but they play it differently.

   - Gita gets to play 10 rounds. Let G be the random variable that counts the number of rounds won by Gita.
   - Sita can keep on playing until she loses 6 rounds. Let S be the random variable that counts the number of rounds won by Sita.

(1 mark) (a) What is the probability $P(G \leq 1)$?

Solution: The variable G is binomially distributed with probability of a single win as $p = 2/5$ and number of trials is 10. So

$$P(G \leq 1) = \binom{10}{0}(3/5)^{10} + \binom{10}{1}(3/5)^9(2/5)$$

(1 mark) (b) What is the mathematical expectation $E(G)$?

Solution: The variable G is binomially distributed as described above. So

$$E(G) = 10 \cdot (2/5) = 4$$

(1 mark) (c) What is the probability $P(S \leq 1)$?

Solution: The variable S is distributed as negative binomial with probability of failure 3/5 and at number of failures allowed as 6. So

$$P(S \leq 1) = \binom{6-1}{0}(3/5)^6 + \binom{1+6-1}{1}(3/5)^6(2/5)$$

(1 mark) (d) What is the mathematical expectation $E(S)$?

Solution: The variable S is distributed according to negative binomial distribution as described above. So

$$E(S) = 6 \cdot \frac{2/5}{3/5} = 4$$

(e) (Bonus) Which of the two has a better chance of winning at least 2 rounds? Justify your answer.

Solution: Comparing the probabilities $P(G \leq 1)$ and $P(S \leq 1)$ we have

$$\frac{P(G \leq 1)}{P(S \leq 1)} = \frac{(3/5)^9 \cdot ((3/5) + 4)}{(3/5)^6 \cdot (1 + 2)} = (3/5)^3 \frac{23}{15} = \frac{23 \cdot 9}{5^4} < 1$$

Thus, $P(G \geq 2) > P(S \geq 2)$.