

**Solutions to Quiz 7**

1. Find any *one* of the decompositions (KAK, KAN or KP) of the following matrix

$$\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$$

**Solution:** We note that the column vectors of the matrix are already orthogonal! Moreover, they both have length  $\sqrt{2}$ . Hence

$$\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} -1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$$

Is the decomposition where the first matrix is real orthogonal (unitary) and the second is positive definite (and also diagonal with positive entries on the diagonal). Hence it is the KP decomposition.

It is also the KAK decomposition with the second K being identity!

It is also the KAN decomposition with N being identity!