Solutions to Quiz 4

1. The mimimal polynomial of a 3×3 matrix A *divides* $T^2(T-1)$. Write down the possible Jordan canonical forms for A.

Solution:

1. The minimal polynomial could be T or $T-1$. In that case A is the 0 matrix or the identity matrix.
2. The minimal polynomial could be $T^2(T-1)$. In that case the Jordan Canonical form is
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$
3. The minimal polynomial could be $T(T-1)$. In this case the characteristic polynomial can be either $T^2(T-1)$ or $T(T-1)^2$. So the matrices are
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
4. The minimal polynomial could be T^2 in which case the characteristic polynomial is T^3 and the matrix is
$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$