Complex Numbers and Quaternions

1. Write the norms, conjugates and inverses of the following complex numbers and quaternions.
   (a) $5 + 12\iota$
   (b) $\sin t + \cos t\iota$.
   (c) $\cos s \cos t + \sin s \sin t\hat{i} + \sin s \cos t\hat{k} + \cos s \sin t\hat{k}$
   (d) $1 + \hat{i} + \hat{j} + \hat{k}$.
2. What are all the quaternions q so that $\hat{i}q\hat{i} = -q$.
3. Given a quaternion (a, v) characterise all quaternions q so that $q \odot (a, v) = (a, v) \odot q$.
4. For a fixed unit vector v define the map $w \mapsto v \times w \times v$ from 3-space to itself. Describe this map in words. Show that your description is correct by calculation.
5. Show that $O(a, v)$ (conjugation by (a, v) on the quaternions of the form $(0, w)$) is a rotation in the plane perpendicular to v by an angle that is determined by a and (v, v).
6. Show that $(a, v) \mapsto O(a, v)$ gives a group homomorphism from S^3 to the group $SO(3)$ of 3 dimensional rotations.
7. Show that this homomorphism is onto and has kernel $\{\pm 1\}$.