Solutions to Quiz 3

1. Convert the following matrix into Smith normal form. At each step write down which row or column operations which you are performing.

$$\begin{pmatrix} 8 & 0 & -6 & -2 \\ 3 & 0 & -3 & 0 \\ -8 & 2 & 4 & -4 \\ 6 & 0 & -6 & 0 \end{pmatrix}$$

Solution:

1. Add 4 times column 2 to column 1
2. Add -2 times column 2 to column 3
3. Add 2 times column 2 to column 4

$$\begin{pmatrix} 8 & 0 & -6 & -2 \\ 3 & 0 & -3 & 0 \\ 0 & 2 & 0 & 0 \\ 6 & 0 & -6 & 0 \end{pmatrix}$$

4. Add 4 times column 4 to column 1
5. Add -3 times column 4 to column 3

$$\begin{pmatrix} 0 & 0 & 0 & -2 \\ 3 & 0 & -3 & 0 \\ 0 & 2 & 0 & 0 \\ 6 & 0 & -6 & 0 \end{pmatrix}$$

6. Add 1 times column 1 to column 3

$$\begin{pmatrix} 0 & 0 & 0 & -2 \\ 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 6 & 0 & 0 & 0 \end{pmatrix}$$

7. Add -2 times row 2 to row 4

$$\begin{pmatrix} 0 & 0 & 0 & -2 \\ 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

8. Interchange column 4 with column 1 (add sign to column 4)

9. interchange column 2 with column 3 (add sign to column 3)

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

10. Interchange column 4 with column 2 (add sign to column 2)

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

11. Add column 1 to column 2.

$$\begin{pmatrix} 2 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

12. Add -1 times row 1 to row 2.

$$\begin{pmatrix} 2 & 2 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

13. Add 2 times column 2 to column 1.

$$\begin{pmatrix} 6 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

14. Add -2 times row 2 to row 1.

$$\begin{pmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

15. Interchange row 1 and row 2 (add sign to row 1)

16. Interchange column 1 and column 2 (add sign to column 1)

17. Interchange row 2 and row 3 (add sign to row 2)

18. Interchange column 2 and column 3 (add sign to column 2)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$