Indian Institute of Science Education and Research Mohali

Differential Equations for Scientists (IDC205) [1]

Academic Session 2016-17

Problem Sheet 03 *Due on* : **August 26, 2016**

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1. Find an integrating factor for the following differential forms

   (a) $y\,dx - x\,dy$.

   (b) $(1 + yxe^x)y\,dx - x\,dy$.

   (c) $(1 + xy)y\,dx + (1 - xy)x\,dy$.

2. Find a family of integral curves for the equation $\dfrac{d}{dx}(y) = \dfrac{y}{x} + \dfrac{x^2 + y^2}{x^2}$.

3. Suppose $\mu(x)$ is an integrating factor for $M(x, y)\,dx + N(x, y)\,dy$; *i.e.* the integrating factor is independent of y. Show that

$$\mu(x) = \exp\left(\int p(x)\,dx\right)$$

   where $p(x) = \dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N}$.

4. Consider the linear differential equation $\dfrac{d}{dx}(y) + P(x)y = 0$. Show that if $f_1, f_2, \cdots, f_n$ are solutions of this equation then so is $\displaystyle\sum_{i=1}^{n} \lambda_i f_i$, where λ_is are arbitrary real numbers. Does the same statement hold for the differential equation $\dfrac{d}{dx}(y) + P(x)y = Q(x)$, where Q is an arbitrary function of x? Justify.

5. Solve the following equation: $x\dfrac{d}{dx}(y) + (x + 1)y = x^3$.

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[1] An interdisciplinary core elective course taught by Amit Kulshrestha during the odd semester of academic session 2016-17 at IISER Mohali.