Indian Institute of Science Education and Research Mohali

Differential Equations for Scientists (IDC205) [1]

Academic Session 2016-17

Problem Sheet 02 *Due on* : **August 16, 2016**

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1. Find a solution of the initial value problem $\frac{d}{dx}(y) = x^2 + y^2;\ y(1) = 3$. Can we use Picard's theorem to ascertain that the initial value problem $\frac{d}{dx}(y) = x^2 + y^2;\ y(1) = 3$ has a solution? Why?

2. Can we use Picard's theorem to ascertain that the initial value problem $\dfrac{d}{dx}(y) = \dfrac{y}{\sqrt{x}};\ y(0) = 1$ has a solution? Why?

3. Use Taylor series to find a family of curves satisfying $\dfrac{d}{dx}(y) = x + y$. Do not forget to comment about the convergence of the series that you get.

4. If $M(x, y)\, dx + M(x, y)\, dy$ is exact then a function $F(x, y)$ satisfying $\frac{\partial}{\partial x}F(x, y) = M(x, y)$ and $\frac{\partial}{\partial y}F(x, y) = N(x, y)$ is called a *solution* of the differential form $M(x, y)\, dx + M(x, y)\, dy$.

   (a) Solve the differential form $(3x^2 + 4xy)\, dx + (2x^2 + 2y)\, dy$.

   (b) Solve the differential form $y \sin(2x)\, dx - \left(y^2 + \cos^2(x)\right)\, dy$.

5. Consider the two differential forms associated to the differential equation $\dfrac{d}{dx}(y) = \dfrac{y}{x}$.

   (a) $y\, dx - x\, dy$,

   (b) $\dfrac{1}{y}\, dx - \dfrac{x}{y^2}\, dy$.

   Show that (b) is exact, while (a) is not. Find a solution $F(x, y)$ of (b). Observe that $F(x, y) = c$ is a family of curves that satisfies the given differential equation.

6. Find orthogonal trajectories to the family of parabolas : $y = cx^2$. Can you identify what these orthogonal curves are?

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[1] An interdisciplinary core elective course taught by Amit Kulshrestha during the odd semester of academic session 2016-17 at IISER Mohali.