End Semester Test

Cosmology and Galaxy Formation (PHY654)
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Duration: 2 hours 30 minutes **Maximum Marks: 40**

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1. Friedmann equations for a universe containing non-relativistic matter and the cosmological constant can be written as:
$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\Omega_{nr}\left(\frac{a_0}{a}\right)^3 + \Omega_\Lambda + (1-\Omega_{nr}-\Omega_\Lambda)\left(\frac{a_0}{a}\right)^2\right]$$
and
$$\frac{\ddot{a}}{a} = H_0^2\left[-\frac{1}{2}\Omega_{nr}\left(\frac{a_0}{a}\right)^3 + \Omega_\Lambda\right]$$
   (a) Show that there are models where $a/a_0 = 0$ is not allowed, i.e., there is a non-zero minimum value of a/a_0. [2]
   (b) What are the values of Ω_{nr} and Ω_Λ for such models? [1]
   (c) Show that there are models where a/a_0 has a maximum allowed value. [2]
   (d) What are the values of Ω_{nr} and Ω_Λ for such models? [1]
   (e) If we consider spatially flat models, i.e., $\Omega_{nr}+\Omega_\Lambda = 1$, then find out the redshift at which matter and cosmological constant contribute equally to energy density of the universe. Evaluate the numerical value if $\Omega_{nr} = 0.2$. [2]
   (f) For spatially flat models, find out the redshift at which the expansion of the universe begins to accelerate. Evaluate the numerical value if $\Omega_{nr} = 0.2$. [2]

2. The luminosity function $\Phi(L)$ is the number density of sources with luminosity between L and $L + dL$. Given a luminosity function
$$\Phi(L)\,dL = \Phi_*\left(\frac{L}{L_*}\right)^{-\alpha}\exp\left[-\left(\frac{L}{L_*}\right)\right]\frac{dL}{L_*} \qquad (1)$$
Here L_*, Φ_* and α are constants. Given the form of the luminosity function above, calculate the following in terms of α, Φ_* and L_*.
   (a) Total number density of galaxies. [2]
   (b) The total luminosity density. [2]
   (c) Mass to light (M/L) ratio of galaxies is known to vary between unity and ten, when measured in terms of the mass to light ratio for the Sun ($M_\odot/L_\odot$). If we take the average mass to light ratio for galaxies to be five times $M_\odot/L_\odot$ then find out the mass density contributed by galaxies. [3]
   (d) If $H_0 = 70$ km/s/Mpc then calculate the numerical value of critical density $\rho_c = 3H_0^2/8\pi G$. [1]
   (e) Calculate the numerical value for the number density of galaxies, luminosity density of galaxies, and mass density of galaxies, and, density parameter of galaxies if $\Phi_* = 5\times10^{-3}$ Mpc^{-3}, $\alpha = 1.1$ and $L_* = 2.4\times10^{10}$ L$_\odot$. [2]

3. The evolution of density contrast δ in linear perturbation theory is given by the following equation:
$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G\bar{\varrho}_{nr}\delta = 0$$
Here, $\bar{\varrho}_{nr}$ is the average density of non-relativistic matter. This is the component which clusters easily at large scales.

(a) In Einstein-deSitter universe, $\Omega_{nr} = 1$ and $\Omega_\Lambda = 0$. Solve Friedmann equations (see problem 1) for this case and find the functional form of $a(t)$. [3]

(b) Solve the equation for density contrast in case of Einstein-deSitter universe and find the two solutions. [3]

(c) Show that $H(t) \equiv \dot{a}/a$ for universes with only non-relativistic matter and cosmological constant satisfies this equation, hence it is a solution of this equation. [4]

4. Consider the universal density profile proposed by Navarro-Frenk-White (NFW).

$$\varrho(R) = \frac{\varrho_{vir} R_{vir}^3}{\left(R_{vir}^2 + R^2\right) R}$$

(a) Given that the average overdensity of the halo up to R_{vir} is $18\pi^2$, and the redshift of collapse if z_{coll}, calculate ϱ_{vir} in terms of the average density of the universe at the redshift of collapse/virialisation. [2]

(b) Express R_{vir} in terms of M_{vir} and the matter density at the redshift of collapse/virialisation. [1]

(c) Solve for the circular velocity as a function of R in this halo. Calculate asymptotic forms at $R \ll R_{vir}$ and $R \simeq R_{vir}$. [2]

(d) Solve for the gravitational potential in the halo as a function of R. [1]

(e) Express virial temperature T_{vir} in terms of the mass of the halo and redshift of collapse. You may assume that the universe is described by the Einstein-deSitter model. For this part, you may assume the halo to be a constant density sphere for the purpose of calculating the binding energy. [2]

(f) If the halo contains dark matter and gas with mass ratio $9:1$, and the gas is in equilibrium at temperature T_{vir} then write down an expression for the density of gas as a function of radius R. You may assume that the gravitational potential is given by the solution of Q.4d and that difference in the density distribution of dark matter and gas does not alter the potential in a significant manner. [2]

Table 1: Useful Numbers

1 parsec	3.08×10^{16} m
G	6.673×10^{-11} m^{3}kg^{-1}s^{-2}
Mass of proton	1.67×10^{-27} kg
$L_\odot$	4×10^{26} W
$M_\odot$	2×10^{30} kg
k_B	1.38×10^{-23} J/K
$\Gamma(0.9)$	1.07
$\Gamma(-0.1)$	10.7