Mid Semester 2

Cosmology and Galaxy Formation (PHY654)

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1. Isothermal halos. An isothermal halo has a density profile $\rho(R) = \rho_{vir} R_{vir}^2 / R^2$. Assume that the background universe is described by the Einstein-deSitter model. Calculate the radius of an isothermal halo of mass M that collapses at redshift z_{coll}. You may use the fact that the over density averaged over the virialized halo at the time of collapse is $18\pi^2$. [3]

2. The gravitational energy of a sphere with radius R and density $\varrho(r)$ is given by:

$$U = -G \int_0^R dr \frac{M(< r)}{r} 4\pi r^2 \varrho dr$$

   Here $M(< r)$ is the mass interior to radius r. Other symbols have their usual meaning. Calculate the gravitational energy for an isothermal halo. [2]

3. Assuming equilibrium, we require that $2\langle KE \rangle = -\langle PE \rangle$, where KE is the kinetic energy and PE is the potential energy, in this case it is the gravitational energy. If the Kinetic energy is expressed in terms of thermal motion of molecules then write down an expression for the (virial) temperature of gas in the halo. [3]

4. Calculate the numerical value of the virial temperature for a halo that has a mass 10^{15} $M_\odot$ and collapses at the present epoch. You may assume that the halo is made of ionized Hydrogen. [2]

5. Consider a one dimensional random walk with equal steps of size l. A step in either the forward or the backward direction is equally likely. Calculate the distribution of displacements after $N \gg 1$ steps. [3]

6. Use the Stirling approximation to derive the *rms* displacement after N steps for the previous problem. [2]

Table 1: Useful Numbers

1 parsec	3.08×10^{16} m
G	6.673×10^{-11} m^{3}kg^{-1}s^{-2}
Mass of proton	1.67×10^{-27} kg
$M_\odot$	2×10^{30} kg
k_B	1.38×10^{-23} J/K