

**Tutorial 5**  
**Cosmology and Galaxy Formation (PHY654)**  
April 22, 2016  
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- Duration of the tutorial session is 60 minutes.
  - You can use your notes, books, online material.
  - You can discuss the problems with your friends and you can ask the instructor for help.
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**NAME:**

**Registration No. :**

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1. Consider an isothermal halo of mass  $M$  that collapses at  $z = 10$  in an Einstein-deSitter universe. The Hubble's constant is  $H_0 = 70$  km/s/Mpc. Find out the virial radius of the halo. [0.5]
2. If the halo has a total angular momentum  $J$ , then obtain a dimensionless form for the angular momentum by using the mass  $M$  of the halo, the binding energy  $E = -GM^2/2R_{vir}$  of the halo, and,  $G$ . The dimensionless form  $\lambda$  is called the spin parameter. [0.5]
3. Find out the virial temperature for the halo. [0.5]
4. If part of the gas cools and collapses to form a rotationally supported disk, find out the scale radius of the disk by requiring conservation of angular momentum, and also that the halo density profile is dominated by dark matter and does not change significantly due to further collapse of gas. [0.5]
5. If the universal gas fraction is  $1/20$  then estimate the gas fraction in the inner part of the halo assuming that the dark matter halo does not shrink at all. How does this depend on the halo spin parameter? [0.5]
6. Calculate the average internal energy density of the gas in the halo when it is at the virial temperature. Average over all the gas in the halo. [0.5]
7. If we assume that the gas in halo is ionized Hydrogen then calculate the cooling time scale for the gas assuming that the cooling rate  $\Lambda(T)$  can be approximated at high temperatures as:

$$\Lambda(T) \simeq 10^{-23} \text{J.m}^3 \cdot \text{s}^{-1} \left( \frac{T}{10^9 \text{K}} \right)^{1/2}$$

Energy loss per unit mass per unit volume per unit time is given by the mean molecular weight times the cooling rate. Define cooling time as the ratio of the internal energy density and the energy loss rate. Obtain an expression for the cooling time. [0.5]

8. Equate the cooling time to the dynamical time of the halo and find out the critical mass for which the cooling time is equal to the dynamical time. [0.5]