Tutorial 4

Cosmology and Galaxy Formation (PHY654)

April 15, 2016

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- Duration of the tutorial session is 60 minutes.
- You can use your notes, books, online material.
- You can discuss the problems with your friends and you can ask the instructor for help.

NAME:
Registration No. :

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1. Junction conditions for a shock are defined as:

$$\begin{aligned} j \equiv \varrho_1 v_{1x} &= \varrho_2 v_{2x} \\ p_1 + \varrho_1 v_{1x}^2 &= p_2 + \varrho_2 v_{2x}^2 \\ \varrho_1 v_{1x} v_{1y} &= \varrho_2 v_{2x} v_{2y} \\ \varrho_1 v_{1x} v_{1z} &= \varrho_2 v_{2x} v_{2z} \\ \varrho_1 v_{1x} \left[h_1 + \frac{1}{2} v_1^2 \right] &= \varrho_2 v_{2x} \left[h_2 + \frac{1}{2} v_2^2 \right] \end{aligned}$$

   Here ϱ_1 is the density of the fluid before the shock, and ϱ_2 is the post-shock density. v_1 and v_2 specify the speed of the fluid, before and after the shock respectively. A further subscript on this denotes a specific component. We have assumed that the shock is in the $y-z$ plane. p_1 and p_2 denote pressure, before and after the shock, respectively. The Enthalpy is denoted by h_1 and h_2.

   (a) Show that we can have a shock where the mass flux j through the discontinuity is zero. This is called a tangential discontinuity. Show that in this case v_y, v_z and ϱ can be discontinuous but not p or v_x. [1]

   (b) For the case where $j \neq 0$, show that v_y and v_z are continuous. Use this to simplify the junction condition for energy flux. [0.5]

   (c) Using the definition of the specific volume: $V_1 = 1/\varrho_1$, $V_2 = 1/\varrho_2$, write down the mass flux in terms of the pressure and specific volume on two sides of the junction. [0.5]

(d) Use this to obtain an expression for the change in the normal component of velocity, i.e., $v_{1x} - v_{2x}$ in terms of the pressure and specific volume on two sides of the shock. [0.5]

(e) Use the above relations to obtain the discontinuity in Enthalpy across the shock in terms of the pressure and specific volume. [0.5]

(f) Write down an expression for discontinuity in the internal energy. [0.5]

(g) If you are given that the equation of state for the gas is $pV^{\gamma} = constant$, $h = \gamma pV/(\gamma - 1)$. Obtain a relation for V_2 in terms of V_1, p_1 and p_2. What is the limiting form if $p_2 \gg p_1$? [0.5]