Solutions to Quiz 8

1. A box contains 3 coins C_1, C_2 and C_3 with probability of head as 1/3, 1/2 and 2/3 respectively. You pick a coin out of the box but you don't know which one it is. You flip the coin 100 times. Justify your answer in each case below with paper and pencil estimates of the likelihoods.

(2 marks) (a) Suppose you get 40 heads. Which coin is the most likely to be the coin that you picked?

(2 marks) (b) Suppose you get 70 heads. Which coin is the most likely to be the coin that you picked?

(1 mark) (c) Suppose you get r heads. Is there a value of r (a positive integer!) for which it will be impossible to decide which coin it is?

Solution: Let $(p_1, p_2, p_3) = (1/3, 1/2, 2/3)$. We have

$$\begin{aligned} L(X=40;p_1)/L(X=40;p_2) &= \frac{2^{160}}{3^{100}} \\ L(X=40;p_2)/L(X=40;p_3) &= \frac{3^{100}}{2^{140}} \\ L(X=40;p_1)/L(X=40;p_3) &= 2^{20} \end{aligned}$$

W can then simplify this

$$\begin{aligned} \frac{2^{160}}{3^{100}} &= \left(\frac{2^8}{3^5}\right)^{20} \\ \frac{3^{100}}{2^{140}} &= \left(\frac{3^5}{2^7}\right)^{20} \end{aligned}$$

Now $\frac{2^8}{3^5} = 256/243 > 1$ and $\frac{3^5}{2^7} = 243/128 > 1$. Hence, $L(X = 70; p_1)$ is the greatest. We have

$$\begin{aligned} L(X=70;p_1)/L(X=70;p_2) &= \frac{2^{130}}{3^{100}} \\ L(X=70;p_2)/L(X=70;p_3) &= \frac{3^{100}}{2^{170}} \\ L(X=70;p_1)/L(X=70;p_3) &= 2^{-40} \end{aligned}$$

W can then simplify this

$$\begin{aligned} \frac{2^{130}}{3^{100}} &= \left(\frac{2^{13}}{3^{10}}\right)^{10} \\ \frac{3^{100}}{2^{170}} &= \left(\frac{3^{10}}{2^{17}}\right)^{10} \end{aligned}$$

Now

$$\frac{2^{13}}{3^{10}} < \frac{2^{14}}{3^{10}} = (128/243)^2 < 1$$

and

$$\frac{3^{10}}{2^{17}} < \frac{3^{10}}{2^{16}} = (243/256)^2 < 1$$

Hence, $L(X = 70; p_3)$ is the greatest.

On order for $L(X = r; p_1)/L(X = r, p_3) = 1$ we must have $r = 50$. In this case, we see that

$$L(X = 50; p_1)/L(X = 50; p_2) = \frac{2^{150}}{3^{100}} = \left(\frac{2^3}{3^2}\right)^{50} = (8/9)^{50} < 1$$

It follows that $L(X = 50; p_2)$ is the greatest.

In order for $L(X = r; p_1)/L(X = r; p_2) = 1$ or $L(X = r; p_1)/L(X = r; p_2) = 1$ we need an identity like $2^a = 3^b$ for *integers* a and b. This is impossible.

It follows that there is no value of r for which the greatest amongst $L(X = r; p_i)$ cannot be found.