Solutions to Quiz 7

1. We repeatedly throw a die. Let X_i denote the random variable that takes the value 5 if the i-th throw is a '1' and -1 if it is anything else. Decide which of the following statements are true and which are false. Justify your answer in each case.

(1 mark) (a) The random variable X_n converges to 0 in probability.

> **Solution:** False. Since $P(|X_n| \geq 1) = 1$, we see that $P(|X_n| > 1/2)$ does not converge to 0 as n goes to infinity.

(1 mark) (b) The random variable $W_n = X_n/n$ converges to 0 in probability.

> **Solution:** True. Since $P(|X_n| < 1/2) = 0$, we see that $P(|W_n| < 1/2k) = 0$ for $n > k$.

(1 mark) (c) The random variable $Y_n = (\sum_{i=1}^{n} X_i)/n$ converges to 0 in probability.

> **Solution:** True. Since $E(X_n) = 0$ and $\sigma^2(X) < \infty$, we have Y_n converges to 0 in probability by the (weak) Law of Large Numbers.

(1 mark) (d) The random variable $Z_n = (\sum_{i=1}^{n} X_i)$ converges to 0 in probability.

> **Solution:** False. If Z_n converges to 0 then so does Z_{n-1} and hence so does $X_n = Z_n - Z_{n-1}$. We have already seen in the first part that this is not so.

(1 mark) (e) The random variable $T_n = (\sum_{i=1}^{n} X_i)/\sqrt{n}$ converges in probability to a random variable Y and Y follows a normal distribution (with suitable mean and variance).

> **Solution:** By the Central Limit Theorem $\sqrt{n}(S_n - \mu)$ converges to a normal distribution with mean 0 and variance $\sigma^2(X)$ for $S_n = (\sum_{i=1}^{n} X_i)/n$ where X_n are independent identically distributed random variables. In our case $\mu = 0$ and $T_n = \sqrt{n}S_n$ by the definition of S_n given above. Hence, an application of the Central Limit Theorem gives the result.