Maximum Likelihood Estimation

1. A box contains 3 coins C_1, C_2 and C_3 with probability of head as 1/2, 2/3 and 1/4 respectively. You pick a coin out of the box but you don't know which one it is. You flip the coin 120 times. Justify your answer in each case below with paper and pencil estimates of the probabilities.

   (a) Suppose you get 70 heads. Which coin is the most likely to be the coin that you picked?

   (b) Suppose you get 90 heads. Which coin is the most likely to be the coin that you picked?

   (c) Suppose you get 40 heads. Which coin is the most likely to be the coin that you picked?

   (d) Suppose you get r heads. Is there a value of r (a positive integer!) for which it will be impossible to decide which coin it is?

2. A child psychologist believes that the intelligence of all children can be classified into 4 categories according to the number of questions that they can answer. Category A answers 1/4 of all questions, Category B answers 1/2 of all questions, Category C answers 3/4 of all questions and Category D answers 9/10 of all questions. A certain child is tested on 100 questions.

   (a) Suppose the child answered 20 questions. Which category would the child be put in.

   (b) Suppose the child answered 40 questions. Which category would the child be put in.

   (c) Suppose the child answered 70 questions. Which category would the child be put in.

   (d) Suppose the child answered 80 questions. Which category would the child be put in.

   (e) Suppose the child answered 85 questions. Which category would the child be put in.

3.

4. In order to check a coin for bias, it was flipped a large number of times.

   (a) 10000 flips resulted in 4819 Heads. What is the most likely estimate for the probability p of a Head with this coin?

   (b) The length and number of "runs" of Heads were also counted and resulted in the following table.

length	0	1	2	3	4	5	6	7	8	9	10	11	12	13
number	2548	1239	623	329	154	75	34	16	9	5	3	1	1	1

   (Here length 0 means we got a Tail on the first flip.) What is the most likely estimate for the probability p of a Head with this coin?

5. A Geiger counter clicks every time an alpha particle is detected. Suppose the gaps between the clicks are (in seconds) given as 6.43, 1.21, 10.41, 4.32, 3.70, 0.55, 5.92. Give an estimate for the frequency of emission of an alpha particle by the radiation source. Justify that this is the most likely value for the frequency.

6. A certain physics experiment results in a random variable X which has probability density $cf(ct)$ in the range $[-1/c, 1/c]$ (and 0 outside) where f is a non-negative differentiable function such that $\int_{-1}^{1} f(t)dt = 1$.

   The value of c is a universal constant but it is not known. Will it be possible to estimate the value of c by doing a large number of experiments? If so, try to find an estimator function.