Revision Assignment

1. We throw a fair die 100 times looking for a '6'.
   (a) Write a formula for the probability that there are there are at least 50 and at most 60 '6's.
   (b) Write an approximate formula for the above probability as an integral.

2. Estimate the following numbers. In each case, first define the number by a series or sequence and then *prove* the relevant convergence.

$$\pi; \log(2); \exp(1); \sqrt{10}; \exp(-0.5)$$

3. We repeatedly flip two fair coins. Let X_i denote the random variable that takes the value 3 if the i-th double flip returns two Heads and -1 if it returns anything else. Which of the following statements are True? Justify your answer.
   (a) The random variable X_n converges to 0 in probability.
   (b) The random variable $W_n = X_n/n$ converges to 0 in probability.
   (c) The random variable $Y_n = (\sum_{i=1}^{n} X_i)/n$ converges to 0 in probability.
   (d) The random variable $Z_n = (\sum_{i=1}^{n} X_i)$ converges to 0 in probability.

4. (a) For what values of a and b can the following be the characateristic function of a random variable?

$$a^2 \cos(t) + b^2 \sin(t)/t$$

   (b) The characteristic function of a random variable X is given by

$$a \sin(t)/t + b \cos(-3t) + c \exp(2t\sqrt{-1})$$

   What are the values of a, b and c for which this has mean 0 and standard deviation 1?