

### Solutions to Quiz 6

- (2 marks) 1. An instructor gives 7 surprise quizzes during a particular semester (which is 14 weeks long). What is the probability that there is no quiz for a period of two weeks?

**Solution:** The expected number of quizzes every two weeks is  $c = 2 \cdot 7/14 = 1$ . Thus, by the Poisson approximation, the probability that there are  $k$  quizzes in a given two week segment is  $e^{-c}c^k/k!$ . For  $k = 0$ , this is  $e^{-c}$  which is  $e^{-1}$ . This is roughly 37% chance of no quiz.

- (3 marks) 2. A chemist is measuring the pH of a certain solution. He wants to have 99% confidence that the result is correct with an error of at most 0.01 after checking 100 samples. What is the largest amount of variance that he can allow?

**Solution:** Assume the bound on the variance is  $M$ . Let  $X$  be the random variable that calculates the average of 100 experiments. We assume that  $E(X) = m$  is the *real* value of pH.

By the weak Law of Large Numbers, we have  $P(|X - m| > c) \leq M/nc^2$ . Here  $n = 100 = 10^2$  is the number of samples and  $c = 0.01 = 10^{-2}$  is the bound on error. We want  $M/nc^2 \leq .01 = 10^{-2}$  to get 99% confidence. So we get  $M \leq 10^{-2+2(-2)+2} = 10^{-4}$ . It is difficult to get such a variance in a real experiment in a real Chemistry lab!