Solutions to Quiz 4

1. Suppose that 10 out of 1000 IISER Mohali students can read Gujarati. You are looking for students who can read a certain document in Gujarati.

(1 mark) (a) If you meet 100 students, write a formula for the probability that *at least* 1 student has managed to read the document.

(2 marks) (b) You make 100 copies of the document and give it to your friends to help you. Assuming no two of your friends ask the same person, write the probability that *exactly* k of your friends report back with success.

(2 marks) (c) What is the expected number of successes in each case?

Solution: The probability of a randomly chosen student reading the document is $p = 10/1000 = 1/100$.

The random variable A counts the number of students out of 100 who have managed to read the document. The A is Binomially distributed, so

$$P(A = 0) = \binom{100}{0} p^0 (1-p)^{100} = (1 - 1/100)^{100}$$

Hence,

$$P(A \geq 1) = 1 - (1 - 1/100)^{100}$$

(Since 100 is "large" this is roughly like $1 - e^{-1}$). The expected number of success in this case is 1 ($= 100p$).

In the second case, the task is divided among 100 friends, and since no two friends meets the same student and there are 10 Gujarati students, the expected probability of successes of each friend is $q = 10/100 = 0.1$.

The random variable B counts the number of *friends* out of 100 who succeed.

$$P(B = k) = \binom{100}{k} q^k (1-q)^{100-k}$$

In this case the expected number of successes is 10 ($= 100c$).