Mid Semester 1

Cosmology and Galaxy Formation (PHY654)

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1. Friedmann equations for a universe containing non-relativistic matter can be written as:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left[\Omega_{nr}\left(\frac{a_0}{a}\right)^3 + (1-\Omega_{nr})\left(\frac{a_0}{a}\right)^2\right]$$

   and

$$\frac{\ddot{a}}{a} = -H_0^2 \frac{1}{2}\Omega_{nr}\left(\frac{a_0}{a}\right)^3$$

   Solve for scale factor as a function of time by requiring that $a = 0$ when $t = 0$ for $\Omega_{nr} > 1$. [3]

   Show that the scale factor varies in proportion with $t^{2/3}$ at early times. [1]

2. Critical density at present is defined as:

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

   Calculate the critical density ρ_c if the Hubble constant is $H_0 = 70$ km/s/Mpc. [1]

3. If Hydrogen atoms contribute 4.5% to the critical density then calculate the number density of protons in the universe. [1]

4. For a universe with only non-relativistic matter and curvature, calculate the density parameter of non-relativistic matter as a function of time, scale factor, or redshift. [3]

5. Show that irrespective of the present value, $|\Omega_{nr}(t) - 1|$ approaches zero at early times. [2]

6. The luminosity function $\Phi(L)$ is the number density of sources with luminosity between L and $L + dL$. Given a luminosity function

$$\Phi(L)\, dL = \Phi_* \left(\frac{L}{L_*}\right)^{-\alpha} \exp\left[-\left(\frac{L}{L_*}\right)\right] \frac{dL}{L_*} \quad (1)$$

   Here L_*, Φ_* and α are constants. Given the form of the luminosity function above, calculate the following in terms of α, Φ_* and L_*.

   (a) Total number density of galaxies. [2]

   (b) The total luminosity density. [2]

Table 1: Useful Numbers

1 parsec	3.08×10^{16} m
G	6.673×10^{-11} m^{3}kg^{-1}s^{-2}
Mass of proton	1.67×10^{-27} kg