Indian Institute of Science Education and Research, Mohali

Cosmology and Galaxy Formation (PHY654)

(January – April 2016)

Problem Set 1

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1. Find the volume of space for the closed Robertson-Walker metric.
2. Show that the Robertson-Walker metric is invariant under translation of the coordinate system.
3. Calculate the critical density ρ_c if the Hubble constant is $H_0 = 70$ km/s/Mpc. If Hydrogen atoms contribute 5% to the critical density then calculate the number density of protons in the universe.
4. Given that the temperature of CMBR is $T_{cmb} = 2.725$ K and it has the black-body spectrum, calculate the number density of CMBR photons and the energy density contributed by these at present. Use the energy density to compute density parameter Ω_{cmb}.
5. Consider a situation where the universe has a fractional anisotropy 10^{-3} and it is thought to have been caused by a uniform magnetic field. Given that the energy density of the universe is approximately 10^{-9} J/m^3, calculate the strength of the magnetic field that gives rise to the observed anisotropy.
6. The luminosity function $\Phi(L)$ is the number density of sources with luminosity between L and $L + dL$. Given a luminosity function

$$\Phi(L)\, dL = \Phi_* \left(\frac{L}{L_*}\right)^{-\alpha} dL \quad ; \quad (L \leq L_*)\,. \tag{1}$$

   Here L_*, Φ_* and α are constants. If these sources are distributed uniformly in space then show that the number of sources brighter than a given flux S varies as $S^{-1.5}$.
7. Given the form of the luminosity function above, calculate the total number density of galaxies and the total luminosity density in terms of α, Φ_* and L_*.
8. Observations show that $\Phi_* = 0.93$ Mpc^{-3}, $\alpha = -1.26$, and the absolute magnitude corresponding to L_* in the R band is $M_* = -21.5$. Calculate L_* and use this to estimate the number density of galaxies and the luminosity density of the universe.
9. If the mass to light ratio of galaxies is ten times the mass to light ratio for the Sun, then calculate the mass density contributed by galaxies. Calculate the density parameter corresponding to this matter density.
10. Solve Friedman equations and obtain the form of scale factor as a function of time for the following cases.
    - A universe dominated by matter and curvature.
    - A universe dominated by matter and cosmological constant.

- A universe dominated by matter and radiation.

11. Compute the age of the universe in the following models:

    - A universe dominated by matter and curvature.
    - A universe dominated by matter and cosmological constant.

    In both these cases, plot the curve for $\tau_{univ} = 12$ Gyr, 13.5 Gyr and 15 Gyr on H_0 — Ω_{nr} plane with $\Omega_{nr} \leq 1$. If the age of the universe is 13.5 Gyr then what values of H_0 and Ω_{nr} are allowed in each case?

12. Consider the FLRW metric. Introduce the following coordinate transformation:

$$d\eta = \frac{dt}{a(t)}$$

    Write down the metric using the conformal time η instead of the cosmological time. Also rewrite the Friedman equations.

13. Calculate the relation between distance (along the light cone) and the conformal time.

14. Calculate the Hubble radius if $H_0 = 70$ km/s/Mpc.

15. Write down the definition of the cosmological horizon. Calculate the cosmological horizon for a universe with $\Omega_\gamma = 10^{-4}$, $\Omega_\Lambda = 0.7$ and $\Omega_{nr} = 1 - \Omega_\Lambda - \Omega_\gamma$.

16. Calculate the horizon size for an observer at $z = 1100$ in the cosmology specified in the last question. Compare it with the Hubble radius at present time.

17. If the universe undergoes exponential expansion starting at $z = 10^{30}$ and last for an increase in scale factor by e^N. How does such an inflationary phase affect the horizon size? What should N be if the horizon at $z = 1100$ be as large as the Hubble radius at present time?