

**Mean, Median, Mode etc**

1. In a population of people in a city, the distribution of heights of individuals (crudely measured using a scaled with least count 2 cm) is given as per the following table:

Height (in cm)	160	162	164	166	168	170	172	174	176	178	180
People (in 1000's)	5	10	25	50	100	120	140	100	50	10	5

$X$  denotes the random variable “the height of a randomly chosen person from the city where each person is equally likely to be chosen”. Calculate the following:

1. The mathematical expectation  $E(X)$  (also called  $\mu(X)$ ).
2. The most likely height.
3. The smallest number  $h$  so that at least 50% of the population has height less than  $h$ .
4. The variance  $\sigma^2(X)$  and the standard deviation  $\sigma(X)$ .

Use a calculator if necessary. Also, try to understand the way in which  $X$  is defined.

2. A multiple choice paper has 10 questions. Each question has 4 choices. The correct answer gets 3 marks and a wrong answer gets  $-1$  mark. A student throws a 4-sided unbiased die to answer each question (the different throws are independent). Let  $X$  be the random variable that denotes the score of the student in the examination.

1. Calculate the expected score  $E(X)$ .
2. Calculate the value of  $X$  for which the probability is the highest.
3. What is the smallest  $s$  so that  $P(X \leq s) \geq 1/2$ ?
4. Calculate the variance  $\sigma^2(X)$ .

Use a calculator if necessary.

3. A die is rolled repeated until we get a 6. The number of rolls is recorded. Let  $X$  denote the random variable that denotes the number of rolls.

1. Calculate the expectation  $E(X)$ .
2. Calculate the value of  $X$  for which the probability is the highest.
3. What is the smallest  $s$  so that  $P(X \leq s) \geq 1/2$ ?
4. Calculate the variance  $\sigma^2(X)$ .

4. Let  $X$  denote the random variable that denotes the sum of the numbers obtained on rolling two dice.

1. Calculate the expectation  $E(X)$ .
2. Calculate the value of  $X$  for which the probability is the highest.

3. What is the smallest  $s$  so that  $P(X \leq s) \geq 1/2$ ?
4. Calculate the variance  $\sigma^2(X)$ .
5.
  - Player A plays a game where he wins 2 rupees with each Head and loses 1 rupee with each Tail;  $X$  is the random variable measuring the money won by A in a single game.
  - Player B plays a game where he wins one rupee with each Head and loses one rupee with each tail but the coin has  $2/3$  chance of getting head;  $Y$  is the random variable measuring the money won by B in a single game.

Calculate and compare the mathematical expectation, median, mode, variance of the variables  $X$  and  $Y$ .