## Chapter 1

## Who Discovered the Pythagorean Theorem?

## 1. Introduction

Poor Pythagoras! That gentle vegetarian ${ }^{1}$ mystic-mathematician would have never imagined that over 2,500 years after his time, hearing his name would have the same effect on some Indians as showing a red rag has on a bull!

At the Indian Science Congress earlier this year, Pythagoras and his theorem were mentioned by many very important persons who went out of their way to make him look like an imposter basking in the lime-light that rightfully belongs to us, the brainy Indians. It is not that Pythagoras doesn't need to be taken down a notch or two, for the evidence that he was the original discoverer of the theorem named after him is simply not there. But that does not by itself mean that the vacated pedestal now belongs exclusively to our own Baudhāyana and his fellow priest-artisans who used ropes to build geometrically complex Vedic altars. And yet, this is exactly what was clearly and repeatedly asserted at the Science Congress.

Here is what the Minister of Science and Technology, Dr. Harsh Vardhan had to say on the matter:

[^0]Our scientists discovered the Pythagoras theorem, but we gave its credit to the Greeks. We all know that we knew bijaganit much before the Arabs, but selflessly we allowed it to be called Algebra. ... whether related to solar system, medicine, chemistry or earth sciences, we have shared all our knowledge selflessly...

The Minister was backed by Dr. Gauri Mahulikar, a Sanskrit scholar from Mumbai University:

In the Śulvasutras, written in 800 BCE, Baudhāyana wrote the geometric formula now known as Pythagoras theorem. It was written by Baudhāyana 300 years before Pythagoras...."2

Between the two of them, the Minister and the Professor proved a theorem dear to the Indian heart, namely: we are not just brainy, but big-hearted as well. We are so big-hearted that we let the likes of Pythagoras to claim priority for what our own Baudhāyana accomplished. We are so big-hearted that we selflessly give away our intellectual riches - from the geometry of Śulvasūtras to advanced mathematical and medical concepts - to the rest of the world. Giving is in what we do.

Compared to the rest of the howlers at the Science Congress - the ancient interplanetary flying machines, the alchemist cows turning grass into gold, for example ${ }^{3}$ - the priority-claim for Baudhāyana has at least one virtue: it is not entirely insane. There is a substantial nugget of truth hidden in an Everest of hype.

There is no doubt that our śulvakaras had indeed mastered the Pythagorean conjecture thoroughly and used it in ingenious ways to create Vedic altars of different areas, while conserving the shapes. They were the first to state it unambiguously. But they were neither alone, nor the first in having this understanding. The first recorded evidence for this conjecture dates back to some 1800 years BCE and it comes from Meso-

[^1]potamia, the present day Iraq. The first proof comes from the Chinese, preempting the Euclidean proof by a couple of centuries, and the Indian proof by at least 1000 years. Even though Pythagoras was not the first to discover and prove this theorem, it does not diminish his achievement. He remains an extremely influential figure not just for history of mathematics, but history of science as well. Pythagoras and his followers were the "first theorists to have attempted deliberately to give the knowledge of nature a quantitative, mathematical foundation". ${ }^{4}$ Giants of the Scientific Revolution, including Johannes Kepler and Galileo Galilei walked in the footsteps of Pythagoras.

In this chapter, we will start with a quick refresher on the Pythagorean Theorem. We will follow this with a straightforward narrative of the different formulations and uses of this theorem, starting with ancient Egypt and Mesopotamia, followed by ancient Greece, India and China. The order is not chronological, and nor does it represent a chain of transmission. While we have evidence of the Greeks getting their start in geometry from the Egyptians and the Mesopotamians, it is quite likely that this conjecture was independently discovered in India and China.

The idea of following the trail of the Pythagorean Theorem from Mesopotamia to China is simply to place ancient India as one among other sister civilizations. It is only through a comparative history of the idea behind this famous conjecture that we will be in a position to judiciously assess India's contribution.

## 2. What is the Pythagorean Theorem?

Before proceeding any further, let us be clear on what the Pythagorean Theorem is all about. Most of us learnt it in middle or high school, but it is a good idea to quickly review it.

The theorem simply states that in a right-angle triangle, the square on the hypotenuse is equal to the sum of the squares on the two sides. (A hypotenuse, to joggle your memory, dear reader, is the longest side of a right-angle triangle which also happens to be the side opposite the right angle).

4 G.E.R. Lloyd, 1970, p. 26.


Figure 1


Figure 2
In figure $1, \mathrm{c}$ is the hypotenuse, while a and b are short and long sides of the right angle triangle, respectively.

So the theorem simply states the following
$c^{2}=a^{2}+b^{2,}$ a relationship that is represented in figure 2 .
This theorem seems simple and intuitive. That is why it has been nominated as a calling-card for the human species to be beamed into
the outer space. ${ }^{5}$ The idea is that any intelligent beings, anywhere in the universe, would recognize its logic - and even perhaps be moved by its beauty. Eli Maor reports that in a 2004 "beauty contest" organized by the journal Physics World, the top winners were Euler's formula, Maxwell's four electromagnetic field equations, Newton's second law, followed by the Pythagorean equation. Not bad for an equation that has been around for more than 3000 years. ${ }^{6}$

It is also one of the most frequently used theorems in all of mathematics. Algebra and trigonometry make use of the equation. Its most obvious and practical use is in the building trade, where it is used for constructing walls perpendicular to the ground, or for constructing perfect squares or rectangles.

This use follows from the fact that the theorem is reversible which means that its converse is also true. The converse states that a triangle whose sides satisfy $\mathrm{a}^{2}+\mathrm{b}^{2}=\mathrm{c}^{2}$ is necessarily right angled. Euclid was the first (1.48) to mention and prove this fact. So if we use lengths which satisfy the relationship, we can be sure that the angle between the short and the long side of a triangle will have to be right angle.

Any three whole numbers that satisfy the Pythagorean relationship and yield a right angled triangle are called Pythagorean triples. The most obvious and the easiest example of these triples is $3,4,5$. That is to say:

$$
\begin{aligned}
& 3^{2}+4^{2}=5^{2} \text { or } \\
& 9+16=25 .
\end{aligned}
$$

That means that any triangle with sides 3,4 and 5 will be a rightangle triangle. As we will see in the rest of this chapter, this method for building right-angle structures was known to all ancient civilizations, not just India. This method is still used by carpenters and architects to get a perfect perpendicular or a perfect square. ${ }^{7}$

[^2]While all right-angle triangles will bear the relationship described by $c^{2}=a^{2}+b^{2}$, not all $a$ and $b$ lengths can be expressed as whole numbers or as ratios of whole numbers. You can see it for yourself: try calculating $c$ for $a=4$ and $b=5$, or $a=7, b=9$. In both cases, you will see that the c cannot be expressed as a whole number. Actually there are only 16 set of whole numbers below 100 that fit into the Pythagorean equation.

There is one particular number for $a$ and for $b$ that puzzled all ancient civilizations that we have records from. That number is one. Imagine a square with side measuring one unit. Now draw a diagonal cutting the square into two right angle triangles.

The simple question is this: how long is the diagonal?


Let us see:
For a right angle triangle, we know that
$c^{2}=a^{2}+b^{2}$
want to locate the corner of the square. On the other side of the corner, draw a line 4 units long, roughly vertical to the first line. Now use the tape to make sure that the edges of the two lengths are exactly 5 units apart. The angle between the two corner lines will be exactly 90 degrees.

In this case,
$c^{2}=1^{2}+1^{2}$
$\mathrm{c}^{2}=2$, therefore $\mathrm{c}=\sqrt{2}$
If you recall your middle-school mathematics, the $\sqrt{ }$ symbol stands for square root. Square root of a number is simply a value which, when multiplied by itself, gives that number.

In the above case, in order to find how long the hypotenuse is, we have to find out square root of two, or in other words, find out that number which, when multiplied with itself will produce the number 2.

Try figuring out the square root of number 2 . You will notice something strange: you simply cannot express the number as a fraction of two whole numbers. What you find is that the decimal fractions of the number that will give you 2 when multiplied by itself simply go on and on, without ending and without repeating themselves. For practical purposes, square root of 2 is taken to be 1.4142136 but the number can go on forever.

Numbers such as these were given the name "alogon" by the Greeks which means "unsayable or inexpressible". We call them irrational numbers.

Irrational numbers were known to all the ancient civilizations that are examined in this chapter. All of them tried to represent these numbers by using rough approximations. Only among the Greeks, however, it led to a crisis of spiritual dimensions. We will shortly explain why, and what they did about it. But we have to start our story from the beginning in Egypt and Mesopotamia.

## 3. Egypt and Mesopotamia

If anyone can take credit for being the first to figure out the Pythagorean Theorem, they have to be the unknown and unnamed builders, landsurveyors, accountants and scribes of ancient Egypt and Mesopotamia (the land we know as Iraq today) sometime between 2000 to 1700 BCE.

Just as ancient India had its śulvakaras who used a length of rope to map out altar designs, ancient Egypt had its harpedonaptai, the "rope stretchers". If Herodotus, the Greek historian who lived in the fifth cen-
tury BCE is to be trusted, these rope-stretchers were surveyors sent out by the pharaohs to measure the farm land for tax purposes every time the river Nile would flood and change the existing boundaries. They are rightly considered the true fathers of geometry, which literally means measurement (metery) of earth (geo): they were the land surveyors sent out by the pharaohs to measure the land for taxation purposes everytime the river Nile would flood and change the existing boundaries.

One would think that a civilization that built the Great Pyramids ${ }^{8}$ would have mastered the right-angle rule and much-much more. Indeed, it has been claimed by Martin Bernal in his well-known book, The Black Athena, that the Greeks learned their sciences and mathematics from Egypt, with its roots in Black Africa. This is not the right forum to resolve this huge controversy, but Bernal's claims regarding the advanced state of mathematics and astronomy in Egypt have been challenged, and are no longer held to be credible by most historians. ${ }^{9}$

The two main mathematical papyri - the Ahmes Papyrus (also called the Rhind Papyrus) that dates back to 1650 BCE and the so-called Moscow Mathematical Papyrus that contains text written some 1850 BCE - don't make any reference to this theorem. While both these papyri contain geometrical problems like calculating the areas of squares, volume of cylinders (for the jars they stored grain in), circumference and areas of circles, the familiar Pythagorean relation is not there. Yet it is hard to imagine how the pyramid makers could have laid the foundations of the square base of pyramid without the familiar $3,4,5$ rule described in the previous section.

A more recent find has thrown new light on this issue: the so-called Cairo Mathematical Papyrus, which was unearthed in 1938 and contains materials dating back to 300 BCE shows that the Egyptians of this, much later era, did know that a triangle with sides $3,4,5$ is right-angled,

8 The best known of them, the Great Pyramid at Gizeh, built around 2600 BCE was the largest building of the ancient world. It rose 481 feet above the ground, with four sides inclined at an angle of 51 degrees with the ground. Its base was a perfect square with an area of 13 acres - equal to the combined base areas of all the major cathedrals in all of Europe. Some 400,000 workers labored on it for 30 years. Burton, 2011, p. 58.
9 See the important paper by Robert Palter (1993) titled, 'Black Athena, Afro-centrism and the History of Science'.
as are triangles with sides $5,12,13$ and 20,21,29. This papyrus contains 40 problems of mathematical nature, out of which 9 deal with the Pythagorean relationship between the three sides of a right triangle. ${ }^{10}$

We may never get the complete story of Egyptian mathematics, as the ancient Egyptians wrote their texts on scrolls made out of flat strips of pith of the papyrus reeds that grew abundantly in the marshes and wetlands of the region. The problem with papyrus is that it is perishable.

But the Mesopotamian civilization that grew not too far away from Egypt on the fertile land between the rivers Tigris and Euphrates in modern-day Iraq is a whole different story in so far historical records go. The clever Sumerians, Assyrians and Babylonians who successively ruled this land have left us a huge library of their literary and mathematical works chiseled on clay tablets which were dried in the sun (and often baked in accidental fires) and are practically indestructible.

As in Egypt, the Mesopotamian mathematics and geometry grew out of administrative needs of the highly centralized state. Temples of local gods and goddesses also needed to keep accounts of the gifts and donations. This led to the flourishing of many scribe-training schools where men (they seem to be all men) learned how to write and do elementary arithmetic. Fortunately for historians, the Mesopotamian people chose a non-degradable material - wet clay that their rivers brought in plenty - to write upon. They used a reed with an edge - quite like our kalam - that could make wedge-shaped marks on the clay. These tablets were then dried in the sun which made them practically indestructible. ${ }^{11}$ Literally thousands of these clay tablets have been recovered and deciphered, including the famous Flood Tablet which tells the story of a great flood, very similar to the Biblical story of the flood and Noah's Ark.

A small fraction of the tablets recovered from schools for scribes contain numerical symbols which were painstakingly deciphered by Professor Otto Neugebauer at Brown University, USA in the 1930s. It is now well-established that the Babylonian people had developed a pretty

[^3]ingenious system that allowed them to use just two symbols - a wedge for the number one and a hook-shaped symbol for the number ten - to represent and manipulate any number, however large. They could do that because they had figured out what is called place value, in which the value of a number changes with the position it occupies. What is more, they also started using a symbol indicating empty space - a forerunner of zero. (Place-value and zero will be examined in the next chapter).

But what is of special interest to us are two tablets which have an iconic status in history of mathematics, namely, Plimpton 322 and a tablet called YBC7289 housed in Columbia and Yale universities, respectively. These tablets reveal that the Mesopotamians knew how to figure out Pythagorean triples, and could also calculate square roots. Some historians conjecture that Plimpton might even be the first record of trigonometry anywhere in the world. ${ }^{12}$

Wikipedia provides a very good description of Plimpton 322:
Plimpton 322 is partly broken clay tablet, approximately 13 cm wide, 9 cm tall, and 2 cm thick. New York publisher George Arthur Plimpton purchased the tablet from an archaeological dealer, Edgar J. Banks, in about 1922, and bequeathed it with the rest of his collection to Columbia University in the mid1930s. The tablet came from Senkereh, a site in southern Iraq corresponding to the ancient city of Larsa. The tablet is believed to have been written about 1800 BC , based in part on the style of handwriting used for its cuneiform script.

A line-drawing of Plimpton 322 (Figure 4a) and a transcript of cuneiform numerals into modern numbers (Figure 4b) are given below. What is written on it that makes it so important? It has four columns of numbers and it appears that there was a fifth column on the left which broken off. The first column from the right is simply a column of serial numbers, from 1-15, while the other three columns contain 15 numbers written in Cuneiform script.

What do these columns of numbers mean? This tablet was first deciphered by Otto Neugebauer and his colleague Alfred Sachs in 1945. Without going into details which can now be found in any standard text book of history of mathematics, they concluded that "the numbers $b$ and $d$ in the second and third columns (from right to left) are

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Figure 4a. Line drawing of Plimpton 322. Source: Eleanor Robson at http://www.dma.ulpgc.es/profesores/pacheco/Robson.pdf

|  | Width | Diagonal |  |
| :--- | :--- | :--- | :--- |
| $1: 59: 00: 15$ | $1: 59$ | $2: 49$ | 1 |
| $1: 56: 56: 58: 14: 50: 06: 15$ | $56: 07$ | $1: 20: 25$ | 2 |
| $1: 55: 07: 41: 15: 33: 45$ | $1: 16: 41$ | $1: 50: 49$ | 3 |
| $1: 53: 10: 29: 32: 52: 16$ | $3: 31: 49$ | $5: 09: 01$ | 4 |
| $1: 48: 54: 01: 40$ | $1: 05$ | $1: 37$ | 5 |
| $1: 47: 06: 41: 40$ | $5: 19$ | $8: 01$ | 6 |
| $1: 43: 11: 56: 28: 26: 40$ | $38: 11$ | $59: 01$ | 7 |
| $1: 41: 33: 45: 14: 03: 45$ | $13: 19$ | $20: 49$ | 8 |
| $1: 38: 33: 36: 36$ | $8: 01$ | $12: 49$ | 9 |
| $1: 35: 10: 02: 28: 27: 24: 26$ | $1: 22: 41$ | $2: 16: 01$ | 10 |
| $1: 33: 45$ | 45 | $1: 15$ | 11 |
| $1: 29: 21: 54: 02: 15$ | $27: 59$ | $48: 49$ | 12 |
| $1: 27: 00: 03: 45$ | $2: 41$ | $4: 49$ | 13 |
| $1: 25: 48: 51: 35: 06: 40$ | $29: 31$ | $53: 49$ | 14 |
| $1: 23: 13: 46: 40$ | 56 | $1: 46$ | 15 |

Figure 4b. Transcription of the Plimpton 322 tablet using modern digits. Source Clark University, Department of Mathematics and Computer Science http:// alepho.clarku.edu/~djoyce/mathhist/plimpnote.html $ᄀ$ )

Pythagorean numbers, this means that they are integer solutions to $d^{2}=b^{2}+l^{2}$ where $d$ and $b$ stand for the diagonal and the leg of the triangle respectively." ${ }^{13}$ To use modern terminology, the numbers tabulated in Plimpton 322 are Pythagorean triples, which as defined in section 2, are whole numbers that fulfill the Pythagorean relation, $\mathrm{a}^{2}+\mathrm{b}^{2}=\mathrm{c}^{2}$.

In other words, Plimpton 322 is the work of some unknown Babylonian mathematician, or a teacher or a scribe trying to find sets of whole numbers which will automatically generate a right angle. What is most striking is that some of the triples listed in the tablet are simply too large for a random, hit-and-trial discovery. ${ }^{14}$ There are many guesses as to how they managed to get these values, but nothing definite can be said about their method.

The second tablet that has received great amount of scrutiny is called YBC 7289, making it the tablet number 7289 in the Yale Babylonian Collection. The tablet dates from the old Babylonian period of the Hammurabi dynasty, roughly 1800-1600 BCE.

This celebrated tablet shows a tilted square with two diagonals, with some marks engraved along one side and under the horizontal diagonal. A line-drawing of the tablet and a sketch in which the cuneiform numerals are written in modern numbers is given below (Figures 5 a and 5 b on the next page):

The number on the top of the horizontal diagonal when translated from the base- 60 of Mesopotamians to our modern 10-based numerals, gives us this number: 1.414213, which is none other than square root of 2 , accurate to the nearest one hundred thousandth. The number below the horizontal diagonal is what we get on multiplying the 1.414213 with the length of the side (30) which, in modern numbers comes to 42.426389. This tablet is interpreted as showing that the Mesopotamians knew how to calculate the square root of a number to a remarkable accuracy.

These two tablets are the first evidence we have of the knowledge of what we today call Pythagorean Theorem.

[^5]

Copyright: A. Aaboe
Figure 5a. Line-drawing of the Yale tablet, YBC 7289.
Source: Mathematical Association of America.


Figure 5b. YBC 7289 transcribed into modern numerals. Source: McTutor History of Mathematics Archives at http://www-history.mcs.st-andrews.ac.uk/index.html

## 4. Pythagoras, the Pythagoreans and Euclid

Pythagoras (about 569 BC -about 475 BC ) is perhaps the most misunderstood of all figures that have come down through history. We all know him as the man who gave us the theorem that - rightly or wrongly - bears his name. But for Pythagoras and his followers, this theorem was not a formula for doubling the square or building precise perpendiculars, as it was for all other civilizations of that time. It is a safe bet that neither Pythagoras nor his followers ever lifted a length of rope, got down on their knees to measure and build anything, for that kind of work was seen fit only for the slaves.

The real - and path-breaking - contribution of Pythagoras was the fundamental idea that nature can be understood through mathematics. He was the first to imagine the cosmos as an ordered and harmonious whole, whose laws could be understood by understanding the ratios and proportions between the constituents. It was this tradition that was embraced by Plato, and through Plato became a part of Western Christianity, and later became a fundamental belief of the Scientific Revolution expressed eloquently by Galileo: "The Book of Nature is written in the language of mathematics."

It is well-recognized that Pythagoras himself was not the original discoverer of the relationship between three sides of a right-angled triangle. Greek accounts written by his contemporaries are very clear that Pythagoras got the idea from the Mesopotamians and perhaps Egyptians, among whom he spent many years as a young man. The words of Sir Thomas Heath, the well-known historian of Greek mathematics, written as long ago as 1921, are apt:

Though this is the proposition universally associated by tradition with the name of Pythagoras, no really trustworthy evidence exists that it was actually discovered by him. ${ }^{15}$

[^6]Neither is there any clear-cut evidence that Pythagoras or his followers offered a proof of the theorem. Those who attribute the proof to Pythagoras cite as evidence stories about him sacrificing a number of oxen when he proved the theorem. Apparently the story about oxen being sacrificed comes from a writer by the name of Apollodorus. But as Thomas Heath has argued, the passage from Apollodorus does mention the sacrifice without mentioning which theorem was being celebrated. The sacrifice story has been challenged on the grounds of the Pythagoreans' strictures against animal sacrifices and meat-eating. ${ }^{16}$

The first Greek proof of the theorem appears in Euclid's classic of geometry called Elements, which was written at least three centuries after Pythagoras. Euclid (around 365 BCE-275 BCE) provides not one, but two proofs of this theorem - theorem 42 in Book I, and theorem 31 of the Book VI. Nowhere does Euclid attribute the proofs to Pythagoras. ${ }^{17}$

Why then did this theorem get Pythagoras' name? No one knows for sure. It is possible that Greeks were following a tradition of attributing new ideas to well-recognized sages - a practice that is very common in Indian scientific and spiritual literature as well. Pythagoras, after all, was no ordinary man: he had a semi-divine status among his followers.

While he did not discover it or prove it, this equation played a most dramatic - one can say, catastrophic - role in Pythagoras' confidence in mathematics and numbers. To understand the catastrophe, one has to understand the fundamental place numbers and ratios occupied in the Pythagorean view of the world.

Pythagoras was a mystic-mathematician, a cross between "Einstein and Mrs. Eddy" to use Bertrand Russell's words. ${ }^{18}$ Or one can say that he was a mystic with a mathematical bent of mind. He saw contemplation of mathematical proportions and ratios as the highest form of meditation that can bring the mind in tune with the Ultimate Reality that he

[^7]believed existed independently of material stuff. What is more, he believed that mathematical knowledge can purify the soul and free it from the cycles of rebirth. (Yes, his spiritual beliefs overlapped with the belief system prevalent in India. More on this below).

Pythagoras was born in 571 BCE (which makes him a rough contemporary of Gautam Buddha in India and Confucius in China) on the island of Samos in the Aegean Sea, just off the coast of modern-day Turkey. He spent many years of his youth in Egypt and later in Mesopotamia. In both places, he immersed himself in the spiritual and mathematical traditions of the host cultures. There is no evidence that he travelled as far east as India, but there is a strong possibility that he picked up the belief in immortality of the soul and its reincarnation from Hindu teachers who were probably present in the courts of Persian kings before Alexander opened a direct line between India and Greece when he came as far as the Indus river in 326 BCE. It was his belief in reincarnation that led him to oppose eating meat and stick to a bean-free vegetarian diet - a dietary practice which is as un-Greek today, as it was then. Like the Hindus, he believed in purification of the soul through contemplation of the Ultimate Reality in order to break the chain of rebirth - except that for him, mathematics was the form that the contemplation of the Ultimate took. ${ }^{19}$

Could he not have picked up the geometry of Baudhāyana and other śulvakaras as well who are estimated to have lived anywhere between 800-300 BCE? It is entirely possible, although the Greek historians of that time have left no record of it. The same historians, on the other hand, have left meticulous records of what he learned from Mesopotamians and Egyptians.

But wherever Pythagoras learned this theorem from, it played a unique role in his philosophy. It led to the discovery of irrational num-

19 For pre-Alexandrian contacts between Indians and Greeks, see McEvilley, 2002, ch. 1. The possibility of Pythagoras learning his beliefs in immortality and rebirth of the soul from Indian philosophers is accepted by many scholars. See Kahn 2001 and McEvilley,2002 for example. One of the many stories that are told about Pythagoras is that he once stopped a man from beating a dog by telling him that he recognized the dog as an old friend, reincarnated. His followers believed that Pythagoras could recall many of his earlier births and that was one reason they treated him as a divine man.
bers (see section 2) which led to a great spiritual crisis for himself and his followers. To understand why a mathematical result would lead to a spiritual crisis, some background is needed.

While we don't have any evidence for Pythagoras discovering the Pythagorean Theorem, his role in discovering the laws of musical sounds is well-attested. It appears that one day as he was walking past a blacksmith's workshop, he was intrigued by the sounds coming from within. So he went in to investigate and found that the longer the sheets of metal that were being hit by the blacksmith's hammer, the lower was the pitch of the sound. When he came back home, he experimented with bells and water-filled jars and observed the same relationship: the more massive an object that is being struck or plucked, the lower the pitch of the sound it produces. He experimented with strings and observed that the pitch of the sound is inversely proportional to the length of the string that is vibrating. He figured out that if a string is plucked at a ratio of 2:1 it produces an octave, 3:2 produces a fifth, 4:3 a fourth.

This was a pivotal discovery - of far greater importance to Pythagoras than the famous theorem he is known for. It made him realize that human experience of something as subjective as music could be understood in terms of numerical ratios: the quality of what pleases the ear was determined by the ratios of the lengths that were vibrating. This was the first successful reduction of quality to quantity, and the first step towards mathematization of human experience. ${ }^{20}$

The realization that what produces music are certain numerical ratios led Pythagoras to derive a general law: that the ultimate stuff out of which all things are made are numbers. Understand the numbers and their ratios and you have understood the Ultimate Reality that lies behind all phenomena, which you can only see in your mind, not through your senses. If all is number - and numbers rule all - then obviously, we should be able to express that number either as whole numbers (integers like 2, 6, 144 etc.) or as fractions of whole numbers (for example, half can be written as one divided by two).

Given how central numbers and numerical ratios were to Pythagoras's view of the unseen reality which humans could access through

[^8]mathematics, one can understand that the discovery that square root of two cannot be expressed as either a whole number or a fraction of two whole numbers would lead to an unprecedented crisis.

This discovery was a direct result of the Pythagorean Theorem. Here is what happened: having understood the right-angled triangle relationship (i.e., $a^{2}+b^{2}=c^{2}$ ) either Pythagoras himself or one of his students tried use it to calculate the diagonal of a square whose side is one unit. They discovered that they simply can't get to a definite number that would terminate somewhere. In other words, they realized that some lengths cannot be expressed as a number. This shattered their fundamental belief that all is number and the ratio of numbers can explain the order of the cosmos.

The legend has it that Pythagoras swore his followers to complete secrecy regarding this awful discovery: they were never to disclose the existence of irrational numbers to anyone. One unfortunate follower by the name of Hippasus who broke the vow of secrecy was pushed to his death from a boat into the Mediterranean Sea - so the story goes.

The crisis led to further developments in Greek mathematics. To begin with, it led to a split between geometry and arithmetic. For Pythagoras, all numbers had shapes. But irrational numbers could not be expressed in shapes. The existence of irrationality was proven later by Aristotle and Euclid.

To conclude this section: yes, Pythagoras was not the original discover of this theorem. But he put it to a different use than it was anywhere else. The truly important discovery of Pythagoras was not the famous theorem, but the laws of music and the existence of irrational numbers.

## 5. Śulvasūtras

We now come to the central theme inspired by the Minister and the Professor mentioned earlier. To recapitulate, they asserted that what the world knows as the Pythagorean Theorem should be renamed after Baudhāyana who discovered it in 800 BCE, which is nearly 200 years before Pythagoras was even born.

As we have already seen, this claim is factually incorrect: there is a great amount of evidence chiseled into the Mesopotamian clay that proves that Pythagoras was already outdone before even Baudhāyana was born! But if we let go of this madness for who came first, we will see that Baudhāyana and his colleagues who lived and worked somewhere between 800 to 500 BCE (or between 600-200 BCE, according to some estimates ${ }^{21}$ ) were extremely creative artisans-geometers in their own right. Their accomplishments don't need to be judged from the Pythagorean or the Greek lens.

What are these Śulvasūtras that we keep hearing about? Who composed them? When? Why? These are some of the questions we will try to answer in this section.

As mentioned earlier, śulva means a cord or string, while sutras are short, poetic sentences that are easy to memorize. These "sutras of the cord" are a part of the kalpa-texts that make up one of the six Vedangas, or limbs of the Vedas, each dealing with a specialized topic ranging from grammar to astronomy. The kalpa literature specializes in ritual matters, including building of fire altars, or vedis, some of them very intricate in shapes and sizes.

The most succinct definition is provided by George Thibaut, the German philologist who first translated these sutras:

> The class of writings, commonly called Śulvasū$t r a s ~ m e a n s ~ t h e ~ " s u t r a s ~ o f ~ t h e ~$ cord". Sulvasūtras is the name given to those portions or supplements of the Kalpasūtras which treat of the measurement and construction of different vedis, or altars, the word sulva referring to the cords which were employed for those measurements. I may remark at once that the sutras themselves don't make use of the word śulva; a cord is regularly called by them rajju (rope). ${ }^{22}$

Out of four extant texts, the two most important are those by Baudhāyana and Āpastamba. Next to nothing is known about these men, but "most likely they were not just scribes but also priest-craftsmen, performing a multitude of tasks, including construction of the

[^9]vedis, maintaining agni and instructing worshippers on appropriate choice of sacrifices and altars." ${ }^{23}$

If it was the need for repeated measurements of land in the floodzones of rivers that gave birth to geometry in Egypt and Mesopotamia, it was the need for precision in Vedic rituals that gave birth to geometry in India. In order for the Vedic yagnas to bear fruit, they had to be carried out precisely according to the guidelines laid out in the Brahmana texts of Yajurveda: the mantras had to be recited just so, the sacrificial animal quartered exactly at specific vertebra, the altar (vedi) for the sacrifice had to be constructed exactly following the prescribed shapes and sizes. Thus, ritual has been recognized as the source of sciences and indeed, by some, of all civilization. ${ }^{24}$ Let us see how the need for exactness in ritual led to advances in geometry in ancient India.

To begin with, the shape of the altar was decided by the goal of the yagna. For example those who desired to go to heaven were required to construct a falcon (syena in Sanskrit)-shaped vedi because as Taittirīya Saṃhitā explained: "the falcon is the best flyer among the birds; and thus he (the sacrificer) having become a falcon himself flies up to the heavenly world." ${ }^{25}$ For those seeking food, the altar should be in the shape of a trough (called drona-cit), while those seeking victory over the enemy were to build an altar in the shape of a rathachakra or a wheel.

What is geometrically challenging about these requirements is this:

- To use Thibaut's words: "every one of these altars had to be constructed out of five layers of bricks... every layer was to consist of 200 bricks [arranged in such a manner] that in all five layers, one brick was never lying upon another brick of the same size and form."
- If this wasn't challenging enough, the area of every altar, whatever its shape - falcon with curved wings, wheel, tortoise,

[^10]trough etc. - had to be equal to $7^{1 / 2}$ square purusha, where a purusha is the height of a man with uplifted arms. ${ }^{26}$

- There was yet another challenge: every-time the sacrifice was carried out after the first construction and consecration, the area had to be increased by one square purusha, until one comes to the one-hundred-and-a-half-fold altar. As Seidenberg explains, "the sacrificer is [symbolically] climbing a ladder, his sacrificial rank being determined by, or determining, the area." ${ }^{27}$
- Here comes the most daunting challenge of all: while the area had to be increased by one square purusha at each subsequent construction, the relative proportions of the single parts had to remain unchanged. In other words, area was to be increased while preserving the shape of the altar.
- There is another twist to altar-making which shows the deep roots of the varna order: If the yajman, or the host of the yagna, was a Brahmin, he was required to set up the sacred fire at eight units east of the household fire, if a prince, eleven and the Vaisya twelve. ${ }^{28}$

Clearly, constructing such altars was no mere "carpentry problem", to use Seidenberg's words, that could be solved with a few "carpenter's rules".29 The technical problems were not trivial, for as Thibaut puts it:

Squares had to be found which would be equal to two or more given squares, or equal to the difference of two given squares; oblongs had to be turned into squares and squares into oblongs; triangles had to be constructed equal to given squares and oblongs and so on....[Even for the most ordinary of vedis] care had to be taken that the sides really stood at right angles, for would the āhavaniya fire have carried up the offerings of the sacrificer to the gods if its hearth had

[^11]not the shape of a perfect square?... [there were also occasions when] a square had to be turned into a circle of the same area. ${ }^{30}$

The most important arsenal in the mental tool-kit of the altar-makers was what we call Pythagorean Theorem.

Baudhāyana gave a very close approximation to this theorem, even though he used four-sided right-angled structures rather than the right-angled triangle that we are familiar with. Here are three sutras (1.9-1.13) from Baudhāyana Śulvasūtras which capture the essence of this theorem, one for the diagonal of a square and another for the diagonal of an oblong or rectangle, followed by Pythagorean triples:
> 1. "The cord which is stretched across in the diagonal of a square (sama-caturasra) produces an area of double the size."

That is: the square of the diagonal of a square is twice as large as the area of the square.
2. a. "The cord stretched on the diagonal of an oblong (dirgha chaturasra) produces both areas which the cords forming the longer and the shorter side of an oblong produce separately."

That is: the square of the diagonal of an oblong is equal to the square on both of its sides. This is an unambiguous statement of the Pythagorean theorem.
2. b. "This (2a) is seen in those oblongs the sides of which are 3 and 4, 12 and 5, 15 and 8,7 and 24,12 and 35,15 and 36 ."

Here, Baudhāyana is enumerating five Pythagorean triangles, that is, right-angled triangles whose sides will yield a hypotenuse, which when squared will yield twice the area of the two sides which have the dimensions described in 2a. All three sides of the resulting triangles can be expressed in whole numbers. ${ }^{31}$

The numbers in 2 b are none other than our old friends, the Pythagorean triples. We encountered them first on Plimpton 322 which dates back at least a thousand years before Baudhāyana. The Pythagoreans not only knew about the triples, but had actually worked out a formula

[^12]for finding these triples. ${ }^{32}$ So we can say that Baudhāyana was no less than his contemporaries, but he was not ahead of them either.

Once these insights were acquired, it became easy to conduct many operations required for altar construction. Thus, doubling the area of a square became a breeze: all you had to do was to figure out the diagonal of the existing square and construct a square on it. Or you could easily triple the size of a square by building an oblong on the diagonal of the second square obtained by doubling the first square.

We also find these principles at work in the construction of a vedi for the soma ritual described by Āpastamba. If one follows Āpastamba's instructions described by Thibaut, it becomes obvious that the altarmakers were using cords and pegs in the ratio of what we would call Pythagorean triples $(5,12,13)$ to construct the east and west side of the vedi at right angles on the axis of the vedi running through the center. ${ }^{33}$

There is lot more to these sutras than just the first enunciation of Pythagoras theorem. Of special interest is the discovery of a procedure for calculating the square roots. The need for calculating the square roots emerged for the same "irrationality" that so bothered the Pythagoreans. The problem is that the diagonal of any square is incommensurable with the length of the sides. This creates a problem for someone who is trying to calculate the diagonal of a square, knowing its sides. We find in Baudhāyana an approximate method of finding square roots, and using this method we get a fairly accurate square root of two to the fifth decimal place. ${ }^{34}$ Here again, our śulvakaras were in good company: the Yale tablet shows the Babylonians knew how to solve the square root of 2 problem, and the Greeks nearly had a mental breakdown over it!

We now come to the controversial matter of proof. For a long time, the mathematical traditions of ancient India and China have been put down as merely "carpenter's rules" which lack proof, while the only

[^13]valid model of proof that is admitted is that modeled on Euclid that proceeds through deductions from first principles. It is true that the authors of Śulvasūtras only meant to convey, in short memorable sutras, how to construct the altars. As a result, they did not try to explain how they arrived at their methods. But that does not mean that the later Indian commentators on these sutras did not feel the need to "remove confusion and doubts regarding the validity of their results and procedures; and to obtain consent of the community of mathematicians." ${ }^{35}$ The Greeks were not the only ones to feel the itch to justify their theorems, albeit the deductive method of proof was unique to them.

Even though Baudhāyana and other śulvakaras don't provide a proof, later texts do. The first Indian proof of the insights regarding right angle and diagonals was provided by Bhaskara who lived in the $12^{\text {th }}$ century.

## 6. "Was Pythagoras Chinese?": the Kou-Ku theorem

Sometime in the $6^{\text {th }}$ century BCE when Pythagoras and his followers were working out their number-based cosmology in islands around the Aegean Sea, when Śulvasūtras were being composed in India, the Chinese, too, had figured out the Pythagorean theorem. Not only that, they had also given an elegant proof for it. Later they would call it kou-ku theorem, which is sometimes also referred to as gou-gu theorem.

The first reference and proof of this theorem appears in the oldest mathematical text known in China. It is called Chou Pei Suan Ching which translates into The Arithmetical Classic of the Gnomon and the Circular Paths of Heaven. Just as in the case of Śulvasūtras, the exact date of this text is not known. To quote from Frank Swetz and T.I.Kao, authors of Was Pythagoras Chinese?:

While the exact date of its origin is controversial, with estimates ranging as far back as 1100 BCE, it can generally be accepted on the basis of astronomical evidence that much of the material in the book was from the time of Confucius, the sixth century BCE and its contents would reflect the mathematical knowledge accumulated in China until that time. ${ }^{36}$

[^14]Chou Pei is largely devoted to using the gnomon to measure the length of the shadow of the sun. ${ }^{37}$ But the first part is devoted to the properties of right-angle triangles. This part consists of a dialogue between Chou Kung (the ruler of Chou) and a wise man by the name of Shang Kao who "knows the art of numbering". Chou Kung wants to know how the astronomers could have "established the degrees of the celestial spheres?" He is puzzled because as he says, "there are no steps by which one may ascent to heavens, and the earth is not measurable by a footrule. I should like to ask you what is the origin of these numbers?"

Shang Kao explains that the art of numbering originates from "the circle and the square. The circle is derived from the square and square from a rectangle." What follows is a statement of what would later be given the name of kou-ku theorem:
let us cut a rectangle diagonally and make the width (kou) 3 units, and the length ( $k u$ ) 4 units. The diagonal (ching) between the two corners will then be 5 units long. ${ }^{38}$

This statement is immediately followed with a proof:
after drawing a square on this diagonal, circumscribe it by half-rectangles like that which has been left outside, so as to form a square plate. Thus the four outer half-rectangles of width 3 , length 4 and diagonal 5 , together make two rectangles (of area 24); then, when this is subtracted from the square plate of area 49 , the remainder is of area 25 . This process is called piling up the rectangles (chi chu).

The methods used by Yu the Great ${ }^{39}$ in governing the world were derived from these numbers.

Chou Kung exclaimed "great indeed is the art of numbering. I would like to ask about the Tao of the use of right-angle triangle."

[^15]After Shang Kao explains the "Tao of the use of right-angle triangle",40 the dialogue ends with Kao declaring:


Figure 6. Hsuan-Thu is considered the earliest proof of Pythagoras Theorem, dating back to around 600 BCE .

[^16]He who understands the earth is a wise man and he who understands the heavens is a sage. Knowledge is derived from a straight line. The straight line is derived from the right angle. And the combination of the right angle with numbers is what guides and rules ten thousand things.

Chou Kung exclaimed: "Excellent indeed." ${ }^{41}$
This dialogue is accompanied by a diagram (figure 6 on page 44). This is what is called hsuan-thu and is considered one of the earliest and most elegant proofs of the hundreds of proofs of the Pythagorean theorem that exist today. ${ }^{42}$

This proof is relevant to the Indian story. As mentioned in the previous section, Śulvasūtras did not provide any proof of the theorem and the first Indian proof appears in the work of Bhaskara in the $12^{\text {th }}$ century.

Some historians believe that Bhaskara's proof is influenced by this ancient Chinese proof. This was first pointed out by Joseph Needham who writes:

Liu Hui [see below] called this figure 'the diagram giving the relations between the hypotenuse and the sum and difference of the other two sides, whereby one can find the unknown from the known.' In the time of Liu and Chao, it was colored, the small central square being yellow and the surrounding rectangles red. The same proof is given by the Indian Bhaskara in the $+12^{\text {th }}$ century.
The Hsuan thu proof of the Pythagoras theorem given in the $+3^{\text {rd }}$ century commentary of Chao Chun-Ching on the Chou Pei is reproduced exactly by Bhaskara in +12 the century. It does not occur anywhere else. ${ }^{43}$

This proof is often confused with Pythagorean proof. But this proof shows an arithmetical-algebraic style of the Chinese which was totally alien to the Greek geometry which abstracted ideal forms from numbers. As Needham puts it, the classic passage from Chou Pei quoted above:
...shows the Chinese arithmetical-algebraic mind at work from the earliest times, apparently not concerned with abstract geometry independent of con-

41 The complete dialogue can be found in Swetz and Kao, pp.14-16, and also in Needham.
42 For a simple explanation of this proof that even those without much mathematical aptitude (including myself) can understand, see http://www.mathisfun.com/ geometry/pthagorean-theorem-proof.html
43 Needham and Wang Ling, 1959, p. 96 and p. 147. This position is supported by Swetz and Kao, p.40, and also finds support from Victor Katz, pp. 240-241.
crete numbers, and consisting of theorems and propositions capable of proof, given only certain fundamental postulates at the outset. Numbers might be unknown, or they might not be any particular numbers, but numbers there had to be. In the Chinese approach, geometrical figures acted as a means for transmutation whereby numerical relations were generalized into algebraic forms. ${ }^{44}$

This theorem shows up again in the $9^{\text {th }}$ chapter of the most wellknown ancient classics of mathematics in China, called Chiu Chang Suan Shu, which translates as Nine Chapters on the Mathematical Art. This work was composed in the Han dynasty ( $3^{\text {rd }} \mathrm{C}$. BCE). The version that survives to the present is a commentary by Liu Hui in 250 CE. Liu Hui has the same iconic stature in China as Aryabhata has in India.

The ninth chapter of the book is titled "Kou-ku" which is an elaboration, in algebraic terms, of the properties of right-angle triangles first described in Chou Pei.

Why kou-ku or gou-gu? To cite Swetz and Kao, in a right angle, the short side adjacent to the right angle is called kou or gou (or "leg"). The longer side adjacent to the right angle is called $k u$ or $g u$ (or "thigh"). The side opposite to the right angle (the hypotenuse) is called hsien (or "bowstring)".45

This chapter contains some kou-ku problems that are famous around the world for their elegance and the delicately drawn sketches that accompany them. These include the so-called "broken bamboo problem" and "the reed in the pond problem". Both of these problems, it is claimed, found their way into medieval Indian and European mathematics texts. The "reed in the pond" problem appears in Bhaskara and the "broken bamboo" in the $9^{\text {th }}$ century Sanskrit classic Ganit Sara by Mahavira. ${ }^{46}$

One thing that the Chinese and the Indian geometers shared - and what set them apart from the Greeks - was that geometry never got

44 Needham and Wang Ling, 1959, pp. 23-24.
45 Swetz and Kao, pp. 26-28.
46 Swetz and Kao, pp. 32-33 for the "reed in the pond" problem, pp. 44-45 for the "broken bamboo" problem. See also Needham, p. 147.

Not having training in mathematics, I am not in a position to render my independent judgement. But there are striking similarities between Li Hui's (250 CE) reed problem and Bhaskara's ( $12^{\text {th }}$ century) Lotus problem. See Swetz and Kao for detailed statement of the problem in both cases.
linked to spiritual and/or philosophical questions, as it did in Greece. It remained more of an art used for practical matters. But that does not mean that their ideas were not supported by arguments and spatial manipulation (as in the Hsuan-thu proof). Just because these proofs did not follow Euclid's axiomatic-deductive methods, does not make them any less persuasive.

## 7. Conclusion

This chapter has followed the trail of the so-called Pythagoras Theorem through centuries, crisscrossing the islands on the Aegean Sea, and traveling through the river valleys of the Nile, the Tigris and the Euphrates, the Indus and the Ganges, and the Yellow River. We have looked at the archeological evidence left behind on Mesopotamian clay tablets and on Egyptian and Chinese scrolls. We have examined the writings of the Greeks and the sutras of our own altar-makers. We have wondered at the achievements of the ancient land-surveyors, builders and mathematicians.

Having undertaken this journey, we are in a better position to answer the question: "Who discovered the Pythagorean Theorem?" The answer is: the geometric relationship described by this theorem was discovered independently in many ancient civilizations. The likely explanation is that the knowledge of the relationship between sides of a right-angle triangle emerged out of practical problems that all civilizations necessarily face, namely, land measurement and construction of buildings - buildings as intricate as the Vedic fire altars, as grand as the Pyramids, as functional as the Chinese dams and bridges, or as humble as simple dwellings with walls perpendicular to the floor.

Where is India in this picture? Indian śulvakaras were one among the many in the ancient world who hit upon the central insight contained in the Pythagorean Theorem: they were neither the leaders, nor laggards, but simply one among their peers in other ancient civilizations. Our Baudhāyana need not displace their Pythagoras, as they were not running a race. They were simply going about their business, Baudhāyana and his colleagues concentrating on the sacred geometry
of fire altars, Pythagoras and his followers worrying about the ratios and proportions that underlie the cosmos.

It is merely an accident of history, undoubtedly fed by the Eurocentric and Hellenophilic biases of Western historians, that the insight contained in the theorem got associated with the name of Pythagoras. Apart from shoring up pride in our civilization, nothing much is to be gained by insisting on a name change. The correct response to Eurocentrism is not Indo-centrism of the kind that was on full display at the Mumbai Science Congress. The correct response is to stop playing the game of one-upmanship altogether.

The itch to be The First is unproductive for many reasons. For one, it turns evolution of science into a competitive sport, history of science into a matter of score- keeping and the historians of science into referees and judges who hand out trophies to the winner. The far greater damage, however, is inflicted on the integrity of ancient sciences and their practitioners whose own priorities and methods get squeezed into the narrow confines of the Greek achievements.

If we really want to honor Baudhāyana and other śulvakaras, a far more sincere and meaningful tribute would be to understand their achievements in the totality of their own context, including the ingenious methods they employed for solving complex architectural problems. Viewing ancient Indian geometry purely, or even primarily, through the lens of Pythagoras actually does a disservice to Baudhāyana, for there is lot more to Śulvasūtras than this one theorem.

It is high time we freed ourselves from our fixation on Pythagoras. Let him Rest in Peace.


[^0]:    1 For reasons that continue to puzzle historians, Pythagoras, who abstained from meat-eating, hated beans. The "Pythagorean diet" was bean-free, as well as meat and fish free. His followers had to swear to follow this diet.

[^1]:    2 See http://www.thehindu.com/news/national/science-congress-lauds-feats-of-ancient-india/article6754106.ece. The priority of ancient priest-craftsmen who composed the Śulvasūtras over Pythagoras has a long history. As early as 1906, Har Bilas Sarda was cheering for Baudhāyana over Pythagoras in his book, Hindu Superiority, pp. 286-287. More recently, Subhash Kak has claimed that the geometry of the Vedic altars contains - in a coded form - advanced astrophysical knowledge such as the exact length of the tropical year and the lunar year, the distance between the sun and the earth, the distance between the moon and the earth in lunar diameters. See Kak, 2005.
    3 India Today has very helpfully listed these howlers. See http://indiatoday.intoday. in/story/5-howlers-from-the-indian-science-congress/1/411468.html.

[^2]:    5 In his classic work of science fiction, From the Earth to the Moon (1865), Jules Verne mentions a German mathematician who suggested that a team of scientists go to Siberia and on its vast plains, set up an enormous, illuminated diagram of Pythagorean theorem so that inhabitants of the Moon would see that we are trying to get in touch. Verne's un-named mathematician has been identified as Carl Friedrich Gauss. See Eli Maor, p. 203.
    6 Maor, p. xii.
    7 If you want to construct a perfect square and you don't have anything but a tape measure and a marker try this: draw a straight line roughly 3 units long where you

[^3]:    10 Burton, 2011, p. 78.
    11 Clay tablets were also recyclable: if a scribe made an error, he could simply knead his tablet into a ball and make a fresh tablet out of it.

[^4]:    12 "If the missing part of the tablet shows up in the future.... Plimpton 322 will go down as history's first trigonometric table." Eli Maor, 2007, p. 11.

[^5]:    13 Otto Neugebauer, p. 37.
    14 For example, row 4 has the following triples 12,709 (the short side), 18,541 the hypotenuse, and 13,500 the third side of a right angle triangle. See Katz, p. 20.

[^6]:    15 Heath, 1921, p. 144. Our esteemed Minister and the Professor were really tilting at windmills. Greeks have always admitted that they learned their geometry from Egyptians and Mesopotamians. All serious historians of mathematics would agree with Sir Heath's words.

[^7]:    16 Heath, 1921, pp. 144-145.
    17 The first proof, I:42, is generally attributed to Eudoxus, who was a student of Plato, while the second proof is attributed to Euclid himself. See Eli Maor, chapter 3.
    18 Mrs. Mary Baker Eddy founded a spiritualist movement called Christian Science in 1879. The quotation is from Russell's well-known History of Western Philosophy, p. 31 .

[^8]:    20 This interpretation is from Arthur Koestler's well-known book, The Sleepwalkers.

[^9]:    21 Agathe Keller (2012) dates Baudhāyana to 600 BCE.
    22 Thibaut 1992[1875], p. 417. George Thibaut is an interesting figure in Indology. He was born in Germany in 1848 and later moved to England to work with Max Muller. In 1875 he became a professor of Sanskrit at Benares Sanskrit College. It was here that he produced his studies on the Śulvasūtras. But his real claim to fame was his work on mimamsa texts. See Keller, 2012, pp. 261-262.

[^10]:    23 George G. Joseph, p. 327.
    24 A. Seidenberg, 1962, proposes that civilization itself has its origin in rituals. We will discuss the contribution of the ritual horse sacrifice (Ashvamedha yagna) in understanding equine anatomy in ancient India in chapter 3.
    25 Thibaut, p. 419.

[^11]:    26 It is not entirely clear how this man of one-purusha height is chosen. Is he any average sized man, or the yajman hosting the yagna?
    27 Seidenberg, 1962, p. 491.
    28 Kim Plofker, p. 24. Plofker calls these units "double-paces where a pace equals 15 angulas". An angula or digit is said to be equal to 14 grains of millet.
    29 Seidenberg is right in poking fun at those Hellenophiles who treat any tradition of geometry that does not justify itself through a Euclidean deduction as merely "carpentry".

[^12]:    30 Thibaut, pp. 420-421.
    31 Thibaut, p. 422-424.

[^13]:    32 The Pythagoreans figured out formulas for calculating triples for an odd number and an even number. These formulas were later given a proof by Euclid. See Katz, p. 38-39 for details.

    33 Thibaut, pp. 424-426.
    34 See Thibaut, pp. 430-431 and Joseph, pp. 334-336 for details. Thibaut provides useful explanations of how śulvakaras could square a circle, build a falcon shaped altar and other complex altars.

[^14]:    35 Srinivas 2008, p. 1833.
    36 Swetz and Kao, 1977, p. 14.

[^15]:    37 Gnomon is a primitive form of a sun-dial. Mesopotamians are known to have used it, the Greeks are known to have borrowed it from Mesopotamians. Indian astronomers knew it as shanku.
    38 Notice the familiar triples 3, 4, 5 .
    39 According to Needham (1959, p. 23), "the legendary Yu was the patron saint of hydraulic engineers and all those concerned with water-control, irrigation and conservancy. Epigraphic evidence from the later Han, when the Chou Pei had taken its present form, shows us, in reliefs on the walls of the Wu Liang tombshrines the legendary culture-heroes Fu-Hsi and Nu-Kua holding squares and compasses. The reference to Yu here undoubtedly indicates the ancient need for mensuration and applied mathematics."

[^16]:    40 Which is nothing more than rules for using a T -square.

