Assignment 3

1. Write a program to perform row reduction of a matrix such that:

   - if the (i, i)-th entry is 0 at i-th stage, it generates an error message and stops.
   - the (i, i)-th entry is used as pivot at the i-th stage.
   - keep track of the operations to produce the LU decomposition.
   - calculate the inverse of M using this method.

   Sage users should make use of the built-in `matrix` type to store the matrix. Others can use the `array` type.

2. Use the built-in random generator to generate a 10×10 matrix M whose entries are uniformly random in the interval $[0, 1]$. Apply your row-reduction program to M to produce its LU decomposition and therefore M^{-1}. Compare $M \cdot M^{-1}$ with identity.

3. For various values of N apply your program to the 2×2 matrix

$$B_N = \begin{pmatrix} (2.0)^{-N} & 1 \\ 1 & 1 \end{pmatrix}$$

   For which values of N is the result "wrong". Why?

4. Modify your program to perform partial pivoting keeping track of the permutation to obtain the πLU decomposition.

5. Apply the partial pivot method to the matrix M and the matrix B_N and compare the results with the previous case.

6. For various values of N apply your programs to the 2×2 matrix

$$C_N = \begin{pmatrix} 2^{-N} & 1 \\ 2^{-N-1} & 2^{-N-2} \end{pmatrix}$$

   For which values of N is the result "wrong". Why?

7. (Starred) For various positive integer values of a, b, c, d try to see when the top-left entry of the matrix

$$E(a, b, c, d) = \begin{pmatrix} 2^{-a} & 2^{(} - b) \\ 2^{-c} & 2^{-d} \end{pmatrix}$$

   is the "correct" choice of pivot. How does this depend on the order between a, b, c and d? Assume that machine epsilon is 2^{-N}, then how does this depend on N? Can you use this to give a way to choose the "correct" choice of pivot?