Affine Schemes:- $X = Sp(\mathcal{O}(X))$ $\quad Sp(k) \uparrow \mathbb{P}^1_k$

$\mathcal{O}(\mathbb{P}^1_k) = k \qquad \mathcal{O}(\mathbb{P}^1_{\mathbb{Z}}) = \mathbb{Z}$

$\mathbb{A}^1 \circlearrowleft x \qquad \mathbb{A}^1 \circlearrowleft x^{-1} \qquad k[x] \cap k[x^{-1}] = k$

$Sp(k[x, x^{-1}])$

"Affine schemes have enough functions" to separate points

$$Sp(R) \to Sp(S) \longleftrightarrow S \to R$$

~~R = k[~~ $S = k[x_1, \dots, x_p]/\langle f_1, \dots, f_q\rangle$

$x_i \to a_i$ s.t. $f_j(a_1, \dots, a_p) = 0 \ \forall j$

$i = 1, \dots, p$

$Sp(S)(R)$

$=$ R-points $=$ solutions in R

$R = k[y_1, \dots, y_a]/\langle g_1, \dots, g_b\rangle$

$x_i \to h_i(y_1, \dots, y_a) \qquad f_j(h_1, \dots, h_p) = 0$

a_i

$y_i \to b_i$ s.t. $g_k(b_1, \dots, b_a) = 0$

$(b_1, \dots, b_a) \in Sp(R)(R)$

$a_i = h_i(b_1, \dots, b_a)$

$$Sp(R) \to Sp(S)$$

1. $S \to R$ 2. R-points of $Sp(S)$ 3. Geom maps $Sp(R) \to Sp(S)$

Arithmetic $\leftrightarrow$

Last lecture:- 1. Q Proj schemes. A

2. $\underline{X \times Y}$ Q Proj

3. $\underline{X \xrightarrow{f} Y}$ s.t. $\Gamma_f \hookrightarrow X \times Y$ closed

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$$\text{Schemes} = \text{Affine schemes} + \text{Patching}$$

$$\text{Manifolds} = \text{Open set in } \mathbb{R}^n + \text{patching}$$

$X =$ "$A(x_1, \dots, x_p; f_1, \dots, f_q)$" $\quad \boxed{S^n}$

$= Sp(\mathbb{Z}[x_1, \dots, x_p]/\langle f_1, \dots, f_q\rangle)$

$X(R) = \{(a_1, \dots, a_p) \mid f_i(a) = 0\}$

$A(x, y; ax^2 + by^2 - 1)$.

$Q(x_1, \dots, x_p; f_1, \dots, f_q; g_1, \dots, g_r)$

$P(x_0, \dots, x_p; f_1, \dots, f_q; g_1, \dots, g_r)(A)$ /Local

$=$ ~~Sheaf~~ "~~Points~~ Locus in $\mathbb{P}^p$ space satisfying $f_1, \dots, f_q$ & not satisfying (weight g) $g_1, \dots, g_r$"

Q-Projective schemes

$$\mathbb{P}^p(A) = \frac{\{ (a_0, \dots, a_p) \mid a_i \text{ is a unit for at least one } i \}}{((a_0,\dots,a_p) \sim (u a_0, \dots, u a_p) \quad u \in A^\times)}$$

not true for A not a UFD

$\mathbb{Z}[\sqrt{-5}] = R$ $\qquad 2 \times 3 = (1+\sqrt{-5})(1-\sqrt{-5})$

$a \cdot b = c \cdot d = 6$

not a p.i.d.

$(2, 1+\sqrt{-5}) \in R^2$ $\qquad R_2[1/2]$

~~$6 \cdot 5 = 2 \cdot 15$~~ $\qquad$ Sp(R)

$\langle 2, 1+\sqrt{-5}\rangle \subsetneq R$

$\mathbb{P}^1(R)$ is not $(a,b)/\sim$

$\mathbb{P}^1(A)$ $\qquad (a;b)$ $\qquad (a,b)$

bad

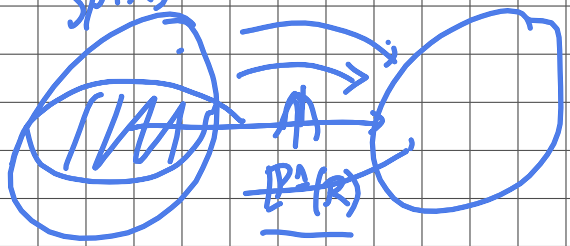
Point of $\mathbb{P}^1(R)$ is described by patching! $\qquad (r_1, r_2)$

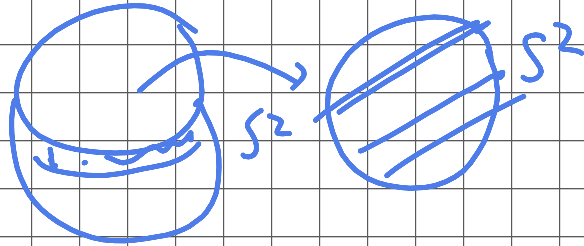
(p_1,p_2) $\quad \mathbb{P}^1(R_a)$ $\qquad \mathbb{P}^1(R_b)$ $\quad (q_1,q_2)$ $\quad \langle a,b\rangle = R.$

$\mathbb{P}^1(R_{ab})$

$(p_1,p_2) = (uq_1, uq_2)$ in $u \in R_{ab}^\times$

Sp(R) = ⊕





"Patching"

$\updownarrow$

~~Ext~~

Gluing

Sp(R) = Sp$(R_a) \cup$ Sp(R_b)

$\mathbb{A}(x,y,z; xy+z^2-1)$ $\qquad xy = (1-z)(1+z)$

$a \cdot b = c \cdot d$

$\mathbb{P}^2 \dots$

*Since ① Affine schemes have non-trivial geometry

② R-points of X are the same as maps. Sp$(R) \to X$

Sp(R) = Sp$(R_{a_1}) \cup \dots$ Sp(R_{a_k})