Barnali: Why does this happen?

Open covers of a scheme

Given a scheme X, suppose $U_i \to X$ is a collection of open subschemes such that $U = \sqcup_i U_i \to X$ is a sheaf-theoretic surjection.

We say that such a collection is a *Zariski open cover* of X.

Let us clarify what this means in the case of an affine scheme $X = \mathrm{Sp}(R)$.

Since each U_i is an open subscheme of $\mathrm{Sp}(R)$, there is an ideal I_i in R such that $U_i = Q(R, I_i)$ is the scheme-theoretic complement of $\mathrm{Sp}(R/I_i)$.

Let J be the ideal in R generated by the ideals I_i as i varies.

If the ideal J is proper, then $\mathrm{Sp}(R/J) \to \mathrm{Sp}(R)$ is an element of $\mathrm{Sp}(R)(R/J)$. However the image of I_i in R/J is $\{0\}$ for all i. Thus, this element is *not* in the image of $\sqcup_i U_i$.

How it is the element of $\mathrm{Sp}(R)(R/J)$?

$\mathrm{Sp}(R)(S) = \mathrm{Hom}(R, S)$

R, S rings.

$R = \mathbb{Z}[x_1, \ldots, x_p]/\langle f_1, \ldots, f_q \rangle$

$\mathrm{Sp}(R)(S) = \{ (a_1, \ldots, a_p) \mid a_i \in S \ \forall i \text{ s.t. } f_j(a_1, \ldots, a_p) = 0 \ \forall j \}$

$$\begin{array}{ccc} \mathbb{Z}[x_1, \ldots, x_p] & & \\ \downarrow & \searrow & \\ R & \longrightarrow & S \end{array}$$

$x_i \longrightarrow a_i$

$R \to R/J$ nat'l hom is an elt. of $\mathrm{Sp}(R)(R/J)$