1. Topology of $\sigma(\mathbb{A}^2) \times \sigma(\mathbb{A}^1)$ is different from $\sigma(\mathbb{A}^2)$

2. So why does $\coprod_{i,j} \sigma(V_{i,j}) \subset \coprod_{i,j} \sigma(U_i) \times \sigma(U_j)$? give an equivalence relation

Topology on $\sigma(\mathbb{A}^1)$ $\quad \cancel{\mathbb{Z}}[X]$

$\mathbb{A}^1_k = Sp(k[X])$ $\quad k$ is a field

irred poly. in $k[X] \longleftrightarrow$ prime ideals.

Topology on $\sigma(\mathbb{A}^1)$: closed sets $\Leftrightarrow$ finite sets

$V(f) \quad f \in k[X]$

$\Rightarrow$ finitely many pts.

"~~Cofinite topology~~" $\quad$ closed sets of finite sets

$\sigma(Sp(\mathbb{Z})) = \{$ prime ideals $: \mathbb{Z}\}$ – "~~Cofinite topology~~"

(0) pointed $\rightarrow$ both (0) is a prime id

"generic point"

$\mathbb{A}^2_k = Sp(k[x,y])$; $\sigma(\mathbb{A}^2_k)$

~~$A[x,y]$, $x+y$)~~ $\quad \wp$

$\sigma(\mathbb{A}^1) = \{ p \mid p \text{ irred} \} \cup \{(0)\}$.

closed sets $\longrightarrow$ Finite subsets of $\wp$

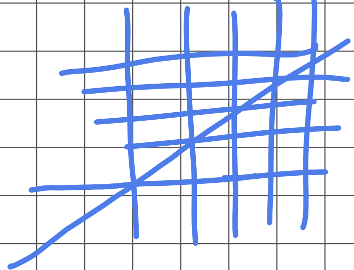
$\longrightarrow \sigma(\mathbb{A}^1)$

$\sigma(\mathbb{A}^2)$ $\quad \langle x+y \rangle$ gives a prime ideal

$\neq$

$\sigma(\mathbb{A}^1) \times \sigma(\mathbb{A}^1)$

$\bigcup_i (U_i \times V_i)$



$$\left[\begin{array}{c} \mathbb{A}^1(T) \times \mathbb{A}^1(T) = \mathbb{A}^2(T) \\ T \times T = T^2 \end{array}\right]$$

$\sigma(X) = \sigma(U)/\sigma(E)$

$U = \coprod_i U_i$

$\sigma E = \coprod_{i,j} V_{i,j}$

$V_{i,j} \longrightarrow U_i \times U_j$

$V_{i,j} \longrightarrow U_i$ or U_j is an open subscheme

$\sigma(V_{i,j}) \longrightarrow \sigma(U_i) \times \sigma(U_j)$ will be a subset!

$\sigma(V_{i,j}) \hookrightarrow \sigma(U_i) \times \sigma(U_j)$

$\sigma(E) \subset \underline{\sigma(U) \times \sigma(U)}$

& gives an equivalence relation

& not $\sigma(\underline{U \times U})$

Zariski topology is internally not Hausdorff for most spaces!

Lecture 11 $\longrightarrow$ re-examine.

① $U, V \subset \mathbb{A}^2$ are non-empty open subschemes

$U \cap V$ is non-empty.

$U = Q(x, y;; \underline{g_1, \dots, g_m})$. $\quad \underline{U \cap V} = Q(x, y;; g_i h_j)$

$V = Q(x, y;; h_1, \dots, h_l)$

$$\mathbb{A}^2 \setminus \{0,0\} \leftarrow \underset{U_1}{\mathbb{A}^2 \setminus x\cdot 2x:=} \amalg \underset{U_2}{\mathbb{A}^2 - y-\langle x\rangle:} = U$$

$$U_1 \cap U_2 = \mathbb{A}^2 \setminus \{xy=0\}$$

$$U \times U = \cancel{\mathbb{A}} \; U_1 \times U_1 \amalg U_1 \times U_2 \amalg U_2 \times U_1 \amalg U_2 \times U_2$$

$$E = \Delta_{U_1} \amalg U_1 \cap U_2 \amalg U_1 \cap U_2 \amalg \Delta_{U_2}$$

$U(R_{u_1}); U(R_{u_2})$

(p_1 , p_2) $\quad E(R_{u_1}, R_{u_2})$

I $\quad p_1 \in U_1(R_{u_1}) \quad p_2 \in U_1(R_{u_2})$

$(p_1, p_2) \in \Delta_{U_1}(R_{u_1,u_2}) = U_1(R_{u_1,u_2})$

$\Rightarrow (p_1 = p_2 \longrightarrow p \in U_1(R) \subset \mathbb{A}^2_R \setminus \{0,0\}$

$\rightarrow$ replace $U_1 \rightarrow U_2$

II $\quad p_1 \in U_1(R_{u_1}) \quad p_2 \in U_2(R_{u_2})$

$(p_1, p_2) \in (U_1 \cap U_2)(R_{u_1,u_2}) \subset U_1 \times U_2$

$\Rightarrow$ Lemma II

$a \in \mathbb{A}^2 \setminus \{0,0\}$

$\rightarrow$ II' $\quad 1 \leftrightarrow 2 \quad U_1 \quad \subset U_2 \times U_1$

$p_i \in U(R_{u_i}) \quad i = 1, \dots, k.$

$(p_i, p_j) \in E(R_{u_i u_j}) \quad i,j = 1..k$

$\rightarrow p \in (U/E)(R).$

$(p, u) \sim (q, v)$

Chapter II - Classification to be posted.